William Stallings
Data and Computer Communications

Chapter 5

Encoding Techniques

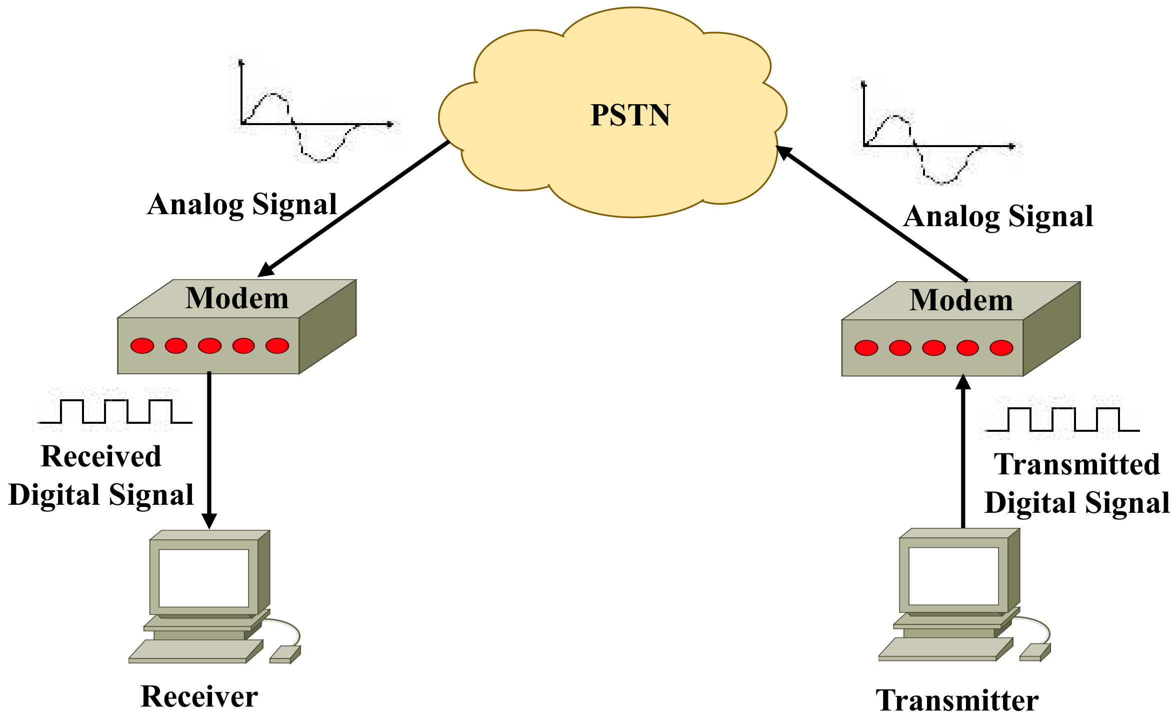
Encoding Techniques

- ⌘Analog data, analog signal
- ⌘Digital data, digital signal
- ⌘Analog data, digital signal
- ⌘Digital data, analog signal

Encoding Techniques

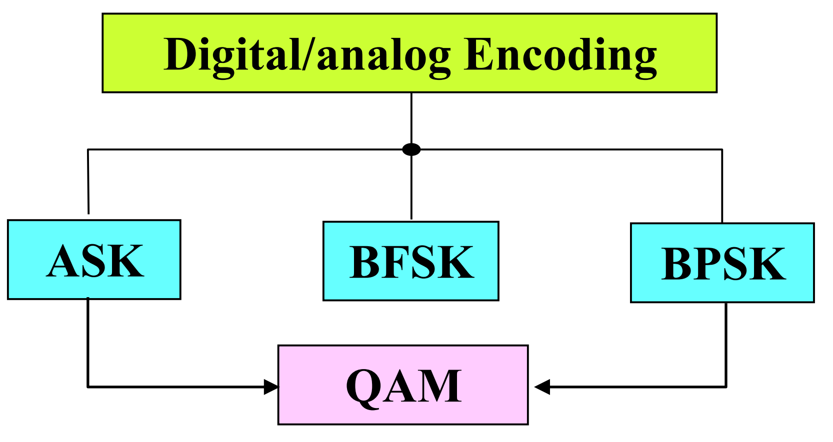
Digital Data, Analog Signal

Modem

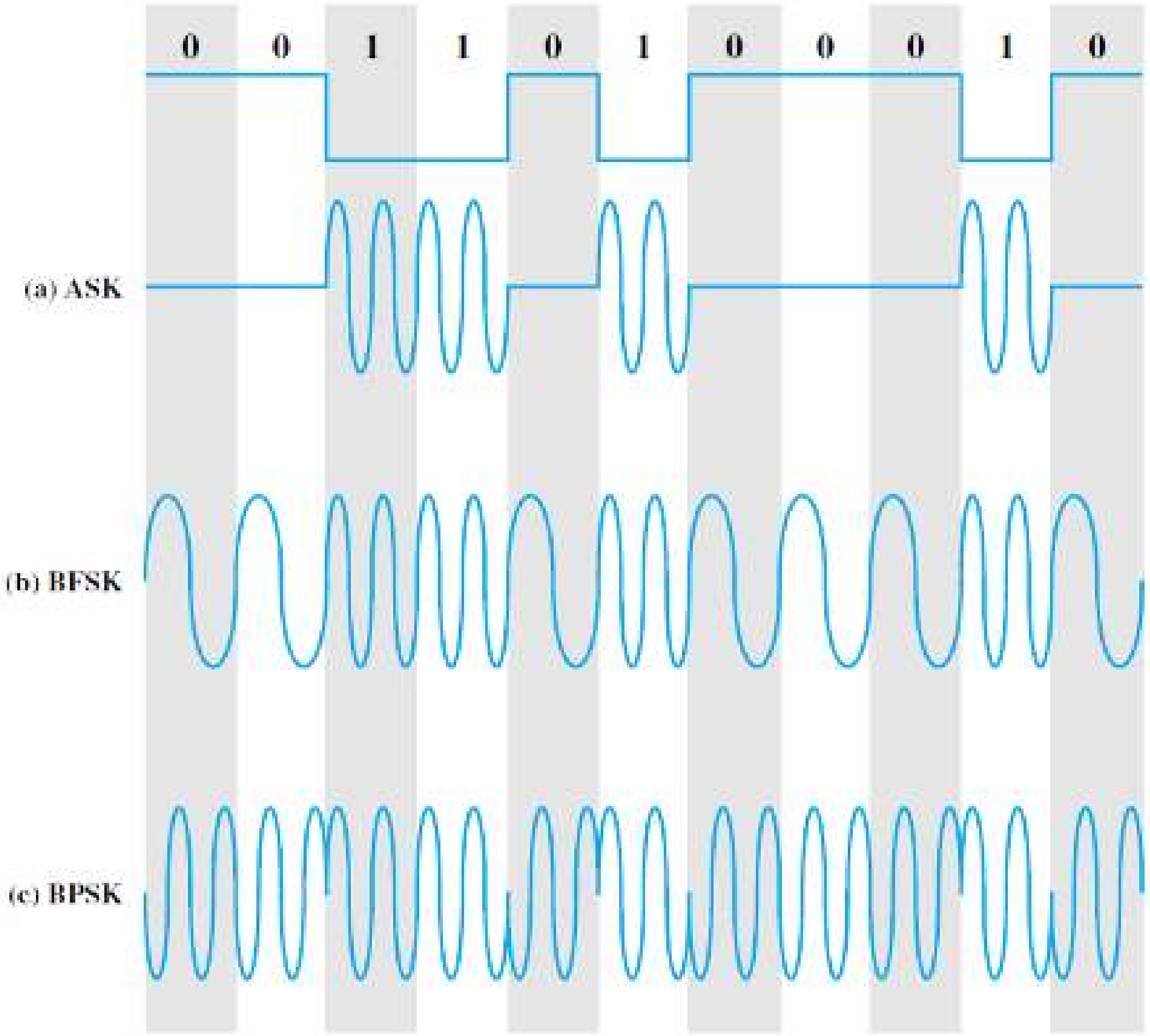

PSTN
Analog Signal
Analog Signal
Modem
Modem
Received
Digital Signal
Transmitted
Digital Signal
Receiver
Transmitter

Digital Data, Analog Signal

- Main use is public telephone system **(PSTN)**
  - has freq range of 300Hz to 3400Hz
  - use **modem** (modulator-demodulator)
- Encoding techniques:
  - Amplitude shift keying (ASK)
  - Binary Frequency shift keying (FSK)
  - Binary Phase shift keying (PSK)

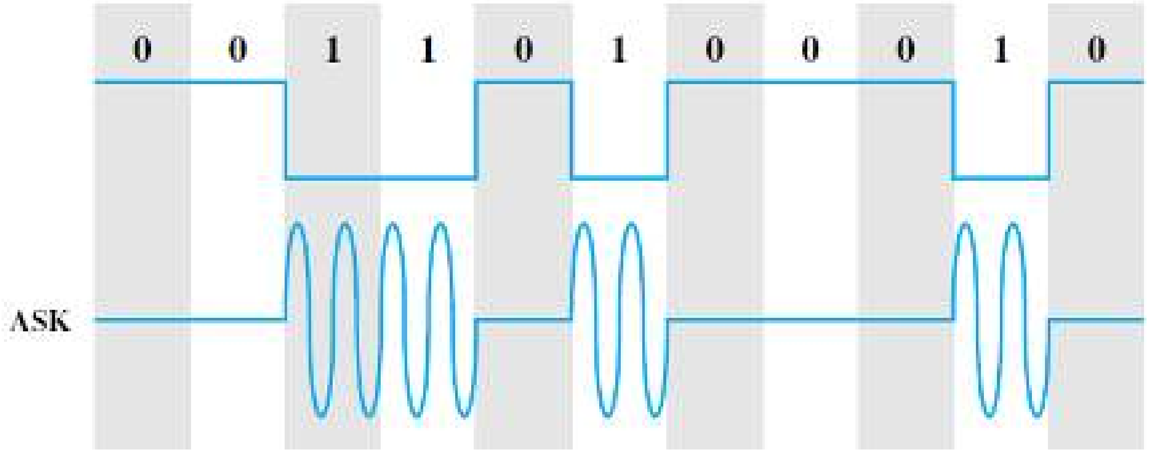


Digital Data, Analog Signals



Amplitude Shift Keying

- Encode 0s and 1s by different carrier amplitudes
  - Usually have one amplitude zero



$$v_{ASK}(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary 1} \\ 0 & \text{binary 0} \end{cases}$$

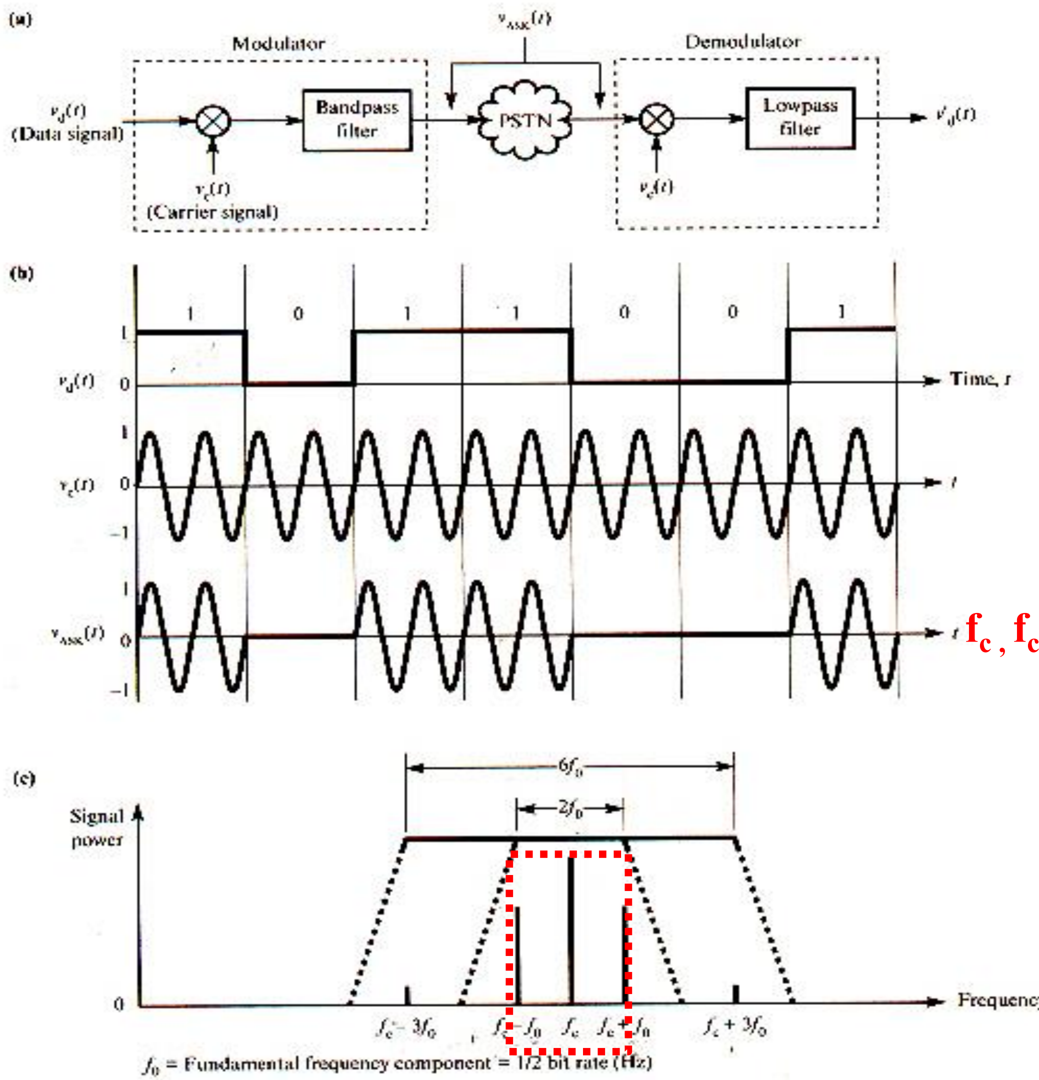
M=2
m=1
R= $mR_s = R_s$
$B_T = (1+r)\ R$
r= 0 → $B_T = R = R_s$

Amplitude Shift Keying

- Inefficient modulation technique
- Used for:
  - up to 1200bps on voice grade lines
  - data transmission in **optical fiber**
    - **LED**
      - 0: absence of light pulse
      - 1: presence of light pulse LED
    - **Laser**
      - 0: low light level
      - 1: higher-amplitude light wave

Amplitude Shift Keying

M=2
m=1
R= mR_s = R_s



Square Wave — f_0, $3f_0$, $5f_0$, …

Carrier — f_c

ASK — f_c , $f_c \pm f_0$, $f_c \pm 3f_0$, $f_c \pm 5f_0$, …

R = R_s = $2f_0$

B_T = (1+r) R

r= 0 → B_T = R = R_s

Amplitude Shift Keying

$$v_d(t) = \frac{1}{2} + \frac{2}{\pi}\{\cos w_0 t - \frac{1}{3}\cos 3w_0 t + \frac{1}{5}\cos 5w_0 t - \ldots\}$$

$$\boldsymbol{\omega_0 = 2\pi f_0}$$

$$v_c(t) = \cos w_c t$$

$$\boldsymbol{\omega_c = 2\pi f_c}$$

$$v_{ASK}(t) = v_c(t).v_d(t)$$

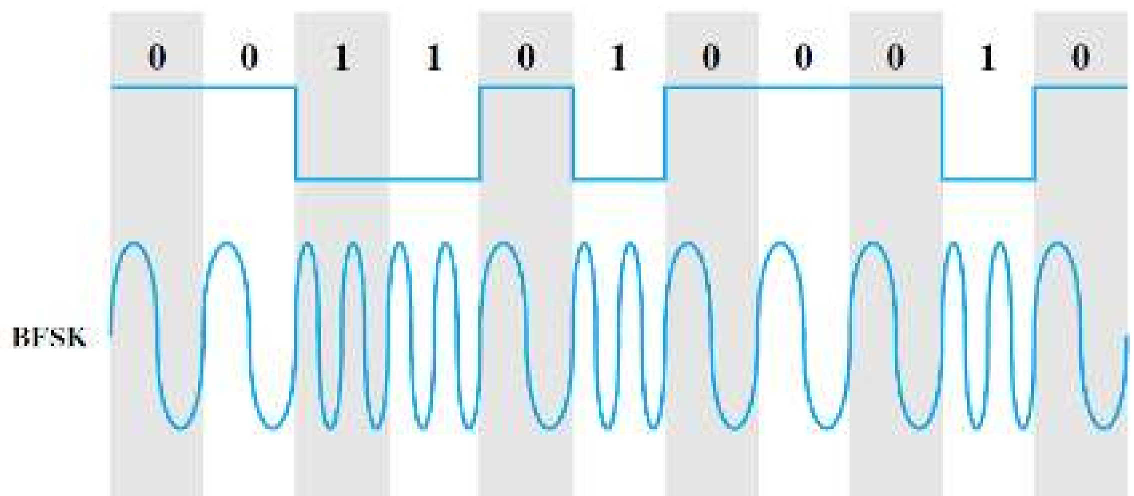
$$v_{ASK}(t) = \frac{1}{2}\cos w_c t + \frac{2}{\pi}\{\cos w_c t.\cos w_0 t - \frac{1}{3}\cos w_c t.\cos 3w_0 t + \frac{1}{5}\cos w_c t.\cos 5w_0 t - \ldots\}$$

$$\begin{aligned} v_{ASK}(t) = \frac{1}{2}\cos w_c t + \frac{1}{\pi}\{&\cos(w_c - w_0)t + \cos(w_c + w_0)t \\ &- \frac{1}{3}\cos(w_c - 3w_0)t - \frac{1}{3}\cos(w_c + 3w_0)t \\ &+ \frac{1}{5}\cos(w_c - 5w_0)t + \frac{1}{5}\cos(w_c + 5w_0)t - \ldots\} \end{aligned}$$

Binary Frequency Shift Keying

- Most common is **binary FSK (BFSK)**
- Two binary values represented by **two different frequencies (near carrier)**

$$v_{FSK}(t) = \begin{cases} A\cos(2\pi f_1 t) & \text{binary 1} \\ A\cos(2\pi f_2 t) & \text{binary 0} \end{cases}$$



M=2
m=1
R= $mR_s = R_s$

$$B_T = \left(\frac{(1+r)M}{\log_2 M}\right)R$$

$$B_T = 2(1+r)R$$

r= 0 → $B_T = 2R = 2R_s$

Binary Frequency Shift Keying

- **FSK** can be represented mathematically as:

$$v_{FSK}(t) = v_{c1}(t).v_d(t) + v_{c2}(t).[1 - v_d(t)]$$

$$where, \quad v_{c1}(t) = \cos 2\pi f_1 t$$

$$v_{c2}(t) = \cos 2\pi f_2 t$$

where, f_1 and f_2 are the two carrier frequencies.

- Less susceptible to error than ASK

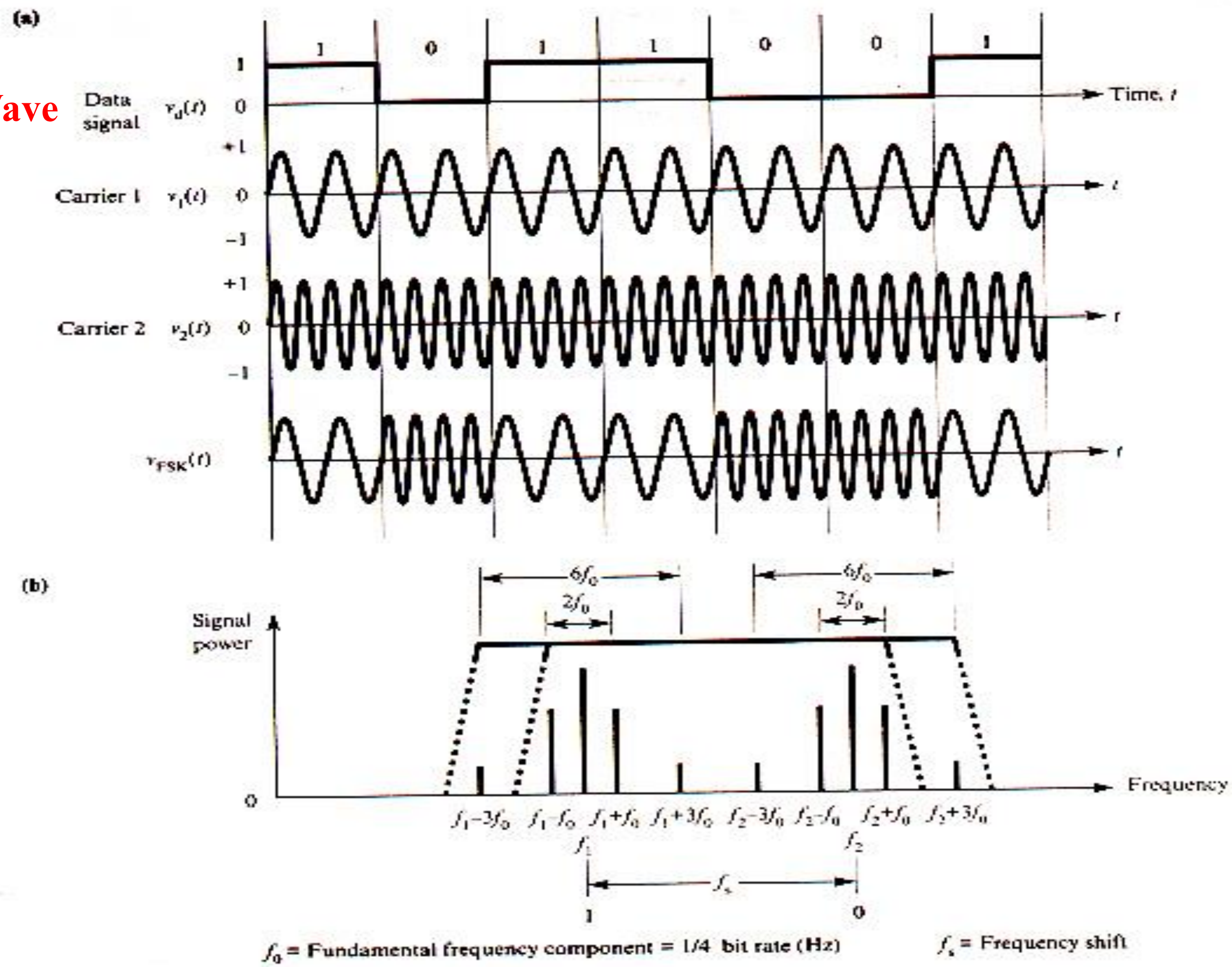
Binary Frequency Shift Keying

Square Wave

Carrier 1

Carrier 2

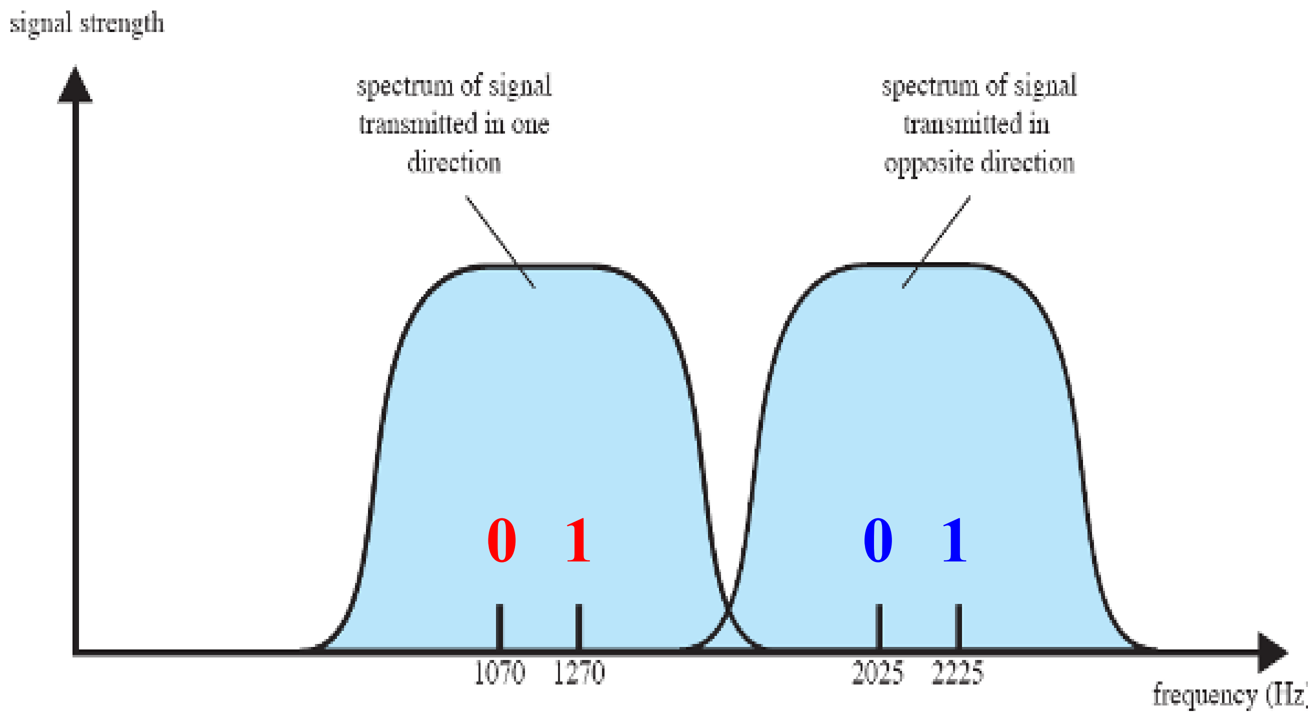
BFSK



Binary Frequency Shift Keying

- An example of use of **FSK for full-duplex** operation over the PSTN.
- The PSTN will pass frequencies in the approximate range 300 to 3400 Hz.
- In one direction, the frequencies used to represent 1 and 0 are centered on **1170 Hz**, with a shift of 100 Hz on either side (i.e., **1070 Hz & 1270 Hz**). The effect of alternating between those two frequencies is to produce a signal.
- Similarly, for the opposite direction, the frequencies used to represent 1 and 0 are centered on **2125 Hz** with a shift of 100 Hz on either side (i.e., **2025 Hz & 2205 Hz**).

BFSK Full-Duplex Transmission



Multilevel FSK

- More than two (M) frequencies used
- Each element represents more than 1 bits
- more bandwidth efficient
- more prone to error
- **For one signal element MFSK**

$$s_i(t) = A\cos(2\pi f_i t), \qquad 1 \le \mathrm{i} \le \mathrm{M}$$

$$f_\mathrm{i} = f_\mathrm{c} + (2i - 1 - M)\, f_\mathrm{d}$$

  - f_c = carrier frequency
  - f_d = difference frequency
  - M = number of different signal elements = 2^m
  - m = number of bits per signal element

Multilevel FSK

$M = 8 \rightarrow m = 3$

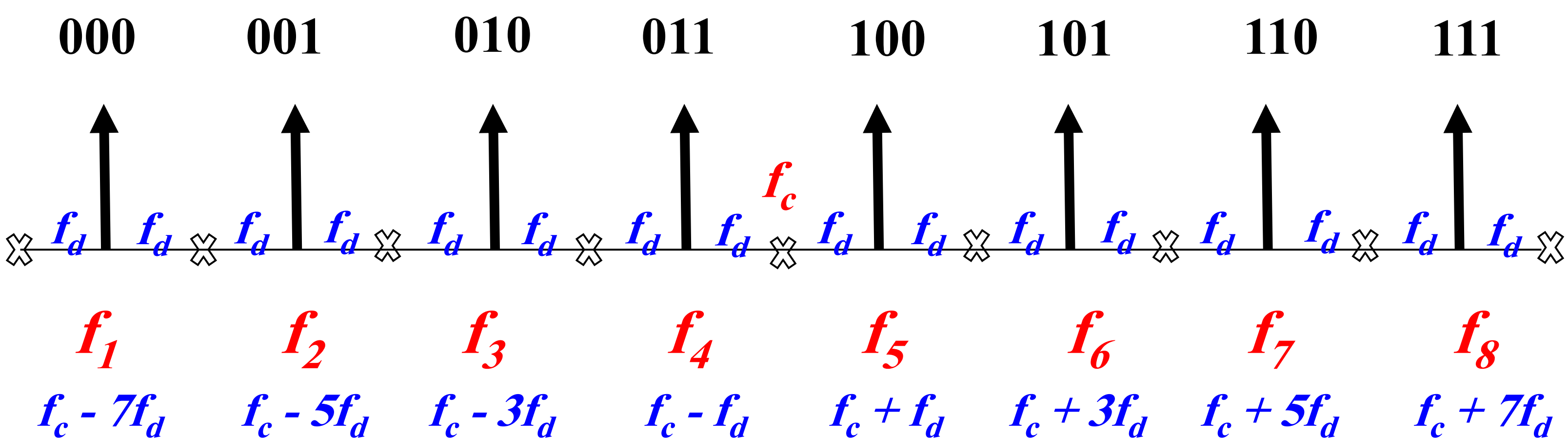
M=8

m=3

R= $mR_s = 3R_s$

$B_T = \left(\frac{(1+r)M}{\log_2 M}\right) R \rightarrow B_T = \frac{(1+r)M}{m} R \rightarrow B_T = (1+r)MR_s$

r= 0 $\rightarrow B_T = MR_s \rightarrow B_T = 2Mf_0$



Multilevel FSK

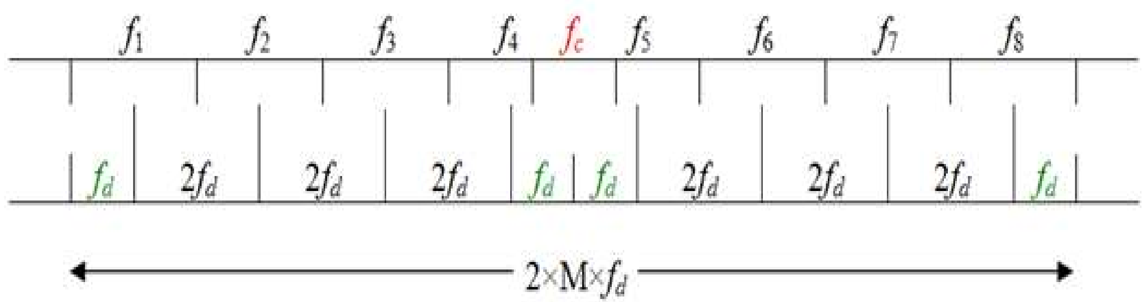
⌘ $\boxed{R = m R_s}$ → $\boxed{1/T_b = m \times 1/T_s}$ → $\boxed{T_b = T_s / m}$

⌘ To match the data rate of the input bit stream, Each output signal element is held for a period of T_s seconds. Thus, one signal element, which is a constant-frequency tone, encodes *m bits.*

⌘ It can be shown that the minimum frequency separation is **$2f_d = 1/T_s$**

→ $\boxed{R_s = 2f_d}$

→ $\boxed{R = 2f_d\,\mathrm{m}}$



⌘ Total bandwidth required (W_d) is:

$$\boxed{W_d = 2M f_d}$$

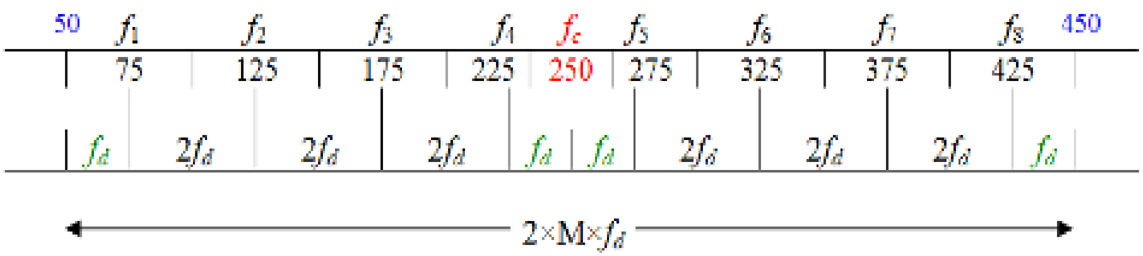
Multilevel FSK Example 1:

- f_c = 250 kHz, f_d = 25 kHz, M = 8 (m = 3)
- Calculate
  - frequency for all 8 possible 3-bit combinations
  - data rate

$f_1 = f_c - 7f_d$	75 KHz	000
$f_2 = f_c - 5f_d$	125 KHz	001
$f_3 = f_c - 3f_d$	175 KHz	010
$f_4 = f_c - f_d$	225 KHz	011
$f_5 = f_c + f_d$	275 KHz	100
$f_6 = f_c + 3f_d$	325 KHz	101
$f_7 = f_c + 5f_d$	375 KHz	110
$f_8 = f_c + 7f_d$	425 KHz	111

Answer:

$$f_i = f_c + (2i - 1 - M)\, f_d$$



$T_s = 1 / R_s$ → T_s = 1/50k = 0.02×10^{-3} second

$T_s = m\,T_b$ → **$T_b = T_s/m$** = 0.02 /3 = 0.0667×10^{-3} second

Data rate supported (R) = m R_s = 3 × 50 kHz = 150kbps

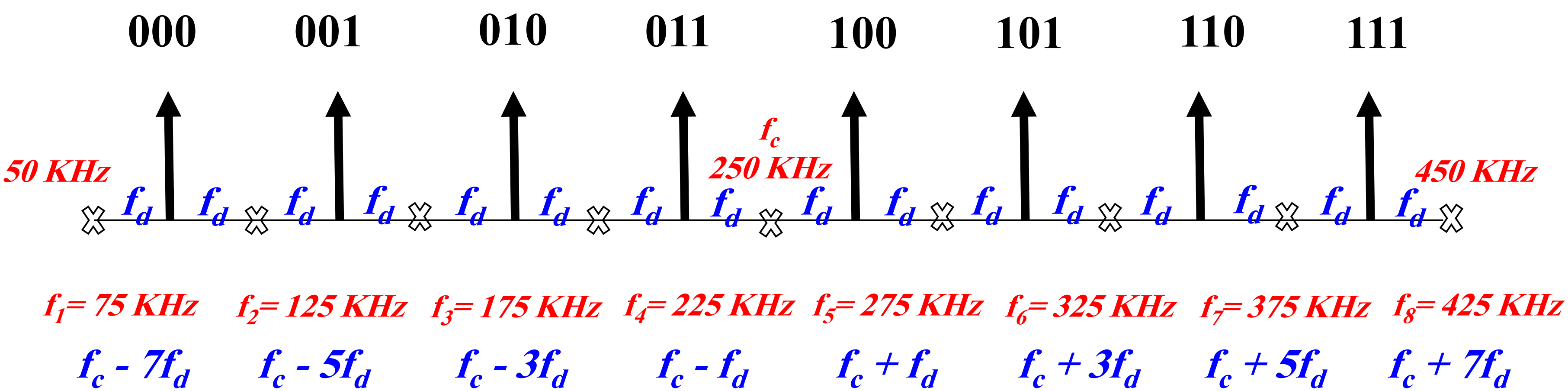
Total Bandwidth (W_d) = M R_s = 8 × 50 kHz = 400 kHz

Multilevel FSK

$M = 8 \rightarrow m = 3$

$f_c = 250$ kHz

$f_d = 25$ kHz



Multilevel FSK Example 1:

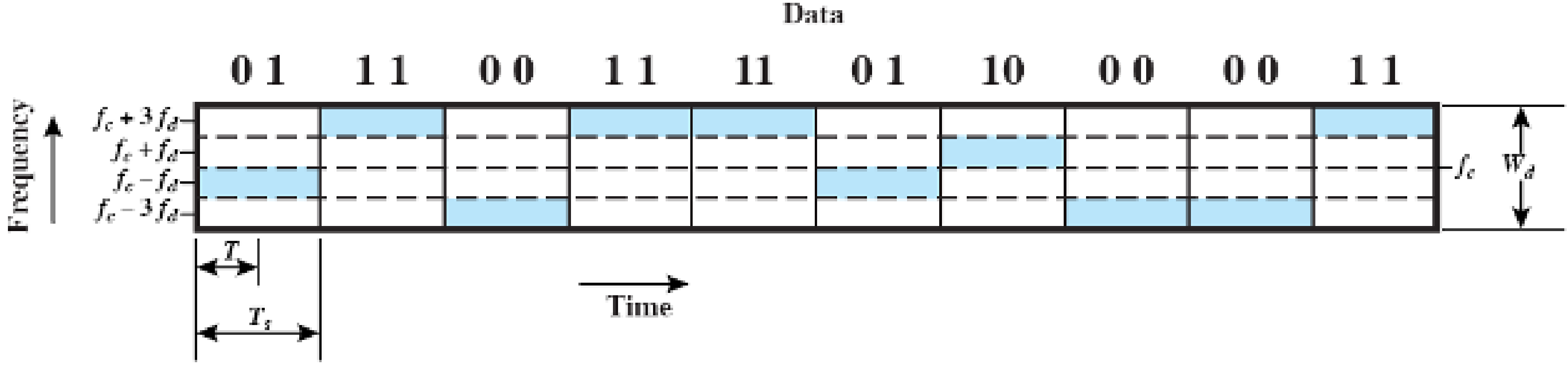
EXAMPLE 5.1 With $f_c = 250$ kHz, $f_d = 25$ kHz, and $M = 8$ ($L = 3$ bits), we have the following frequency assignments for each of the eight possible 3-bit data combinations:

$f_1 = 75$ kHz	000	$f_2 = 125$ kHz	001
$f_3 = 175$ kHz	010	$f_4 = 225$ kHz	011
$f_5 = 275$ kHz	100	$f_6 = 325$ kHz	101
$f_7 = 375$ kHz	110	$f_8 = 425$ kHz	111

This scheme can support a data rate of $1/T = 2Lf_d = 150$ kbps.

Multilevel FSK Example 2:

- MFSK with M = 4
- Bit stream of 20 bits, encoded 2 bits at a time
- Each of the 4 combinations use different frequency
- Column = Ts, row = frequency used



Multilevel FSK Example 2:

EXAMPLE 5.2 Figure 5.9 shows an example of MFSK with $M = 4$. An input bit stream of 20 bits is encoded 2 bits at a time, with each of the four possible 2-bit combinations transmitted as a different frequency. The display in the figure shows the frequency transmitted (y-axis) as a function of time (x-axis). Each column represents a time unit T_s in which a single 2-bit signal element is transmitted. The shaded rectangle in the column indicates the frequency transmitted during that time unit.

Binary Phase Shift Keying

- phase of carrier signal is shifted to represent data

$$v_{BPSK}(t) = \begin{cases} A\cos(2\pi f_c t) \\ A\cos(2\pi f_c t + \pi) \end{cases} = \begin{cases} A\cos(2\pi f_c t) & \text{binary 1} \\ -A\cos(2\pi f_c t) & \text{binary 0} \end{cases}$$

M=2

m=1

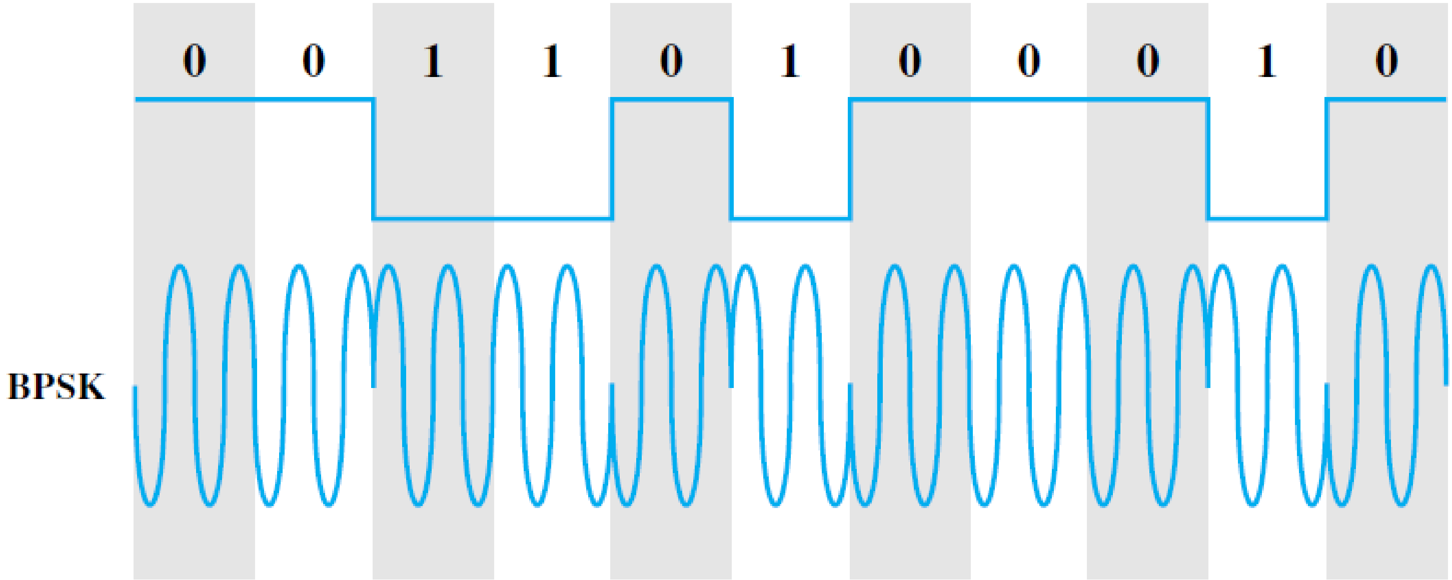
R= mR_s = R_s

$$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$$

$$B_T = (1+r)R$$

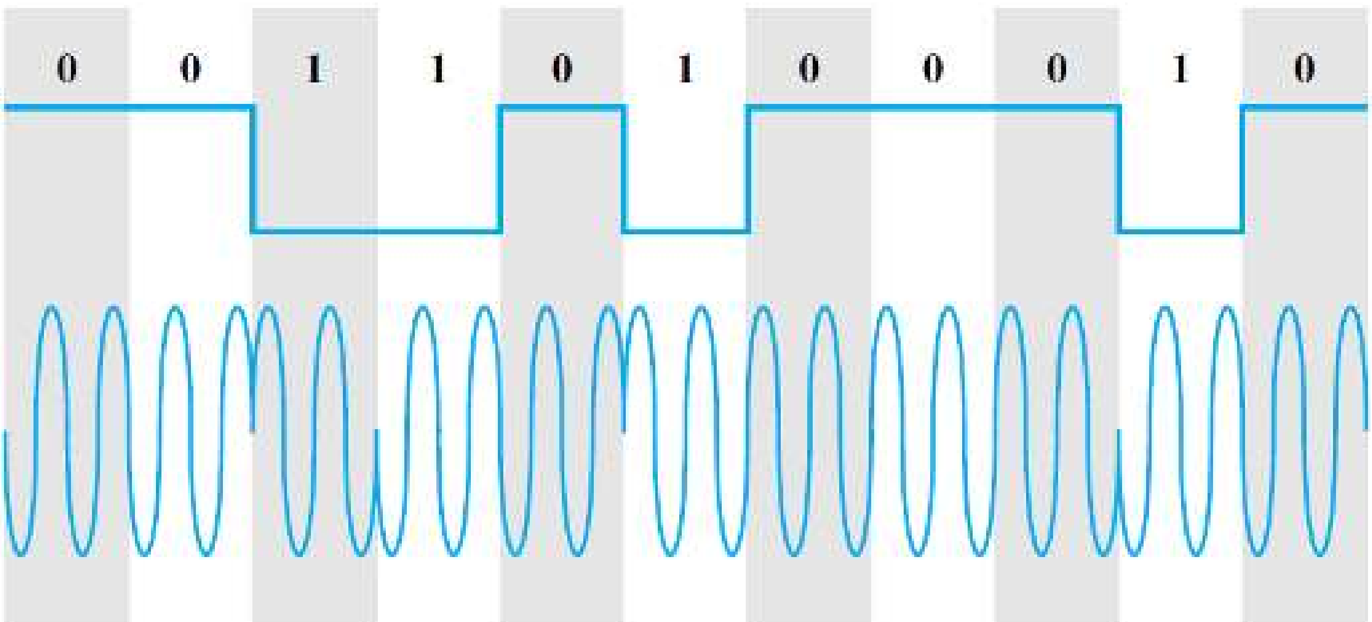
r= 0 → B_T = R = R_s

- **Binary PSK**
  - Two phases represent two binary digits



Binary Phase Shift Keying

- **Differential PSK**
  - Phase shifted relative to previous transmission rather than some reference signal



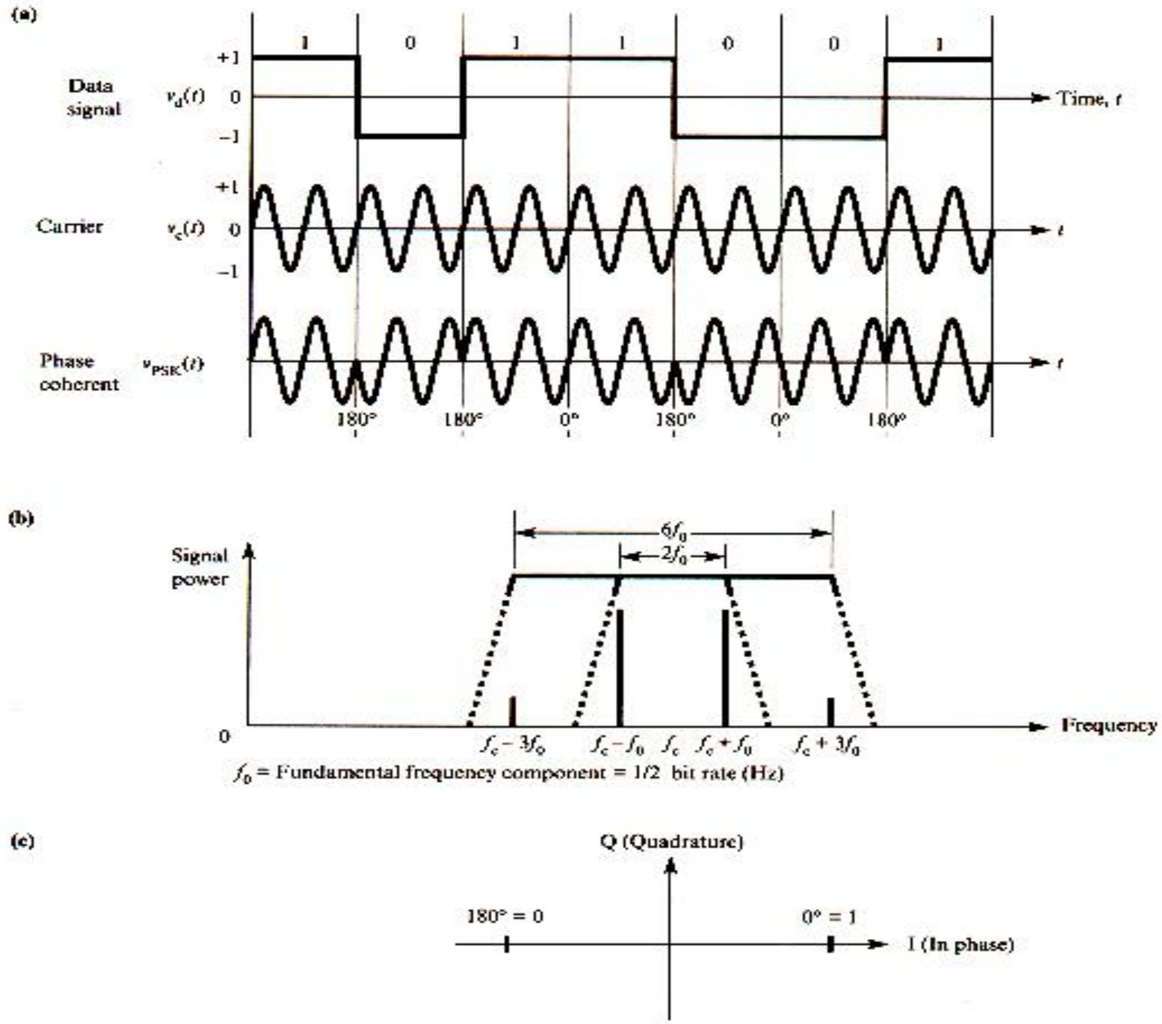
Differential Phase-Shift Keying (DPSK)

Binary Phase Shift Keying

Square Wave

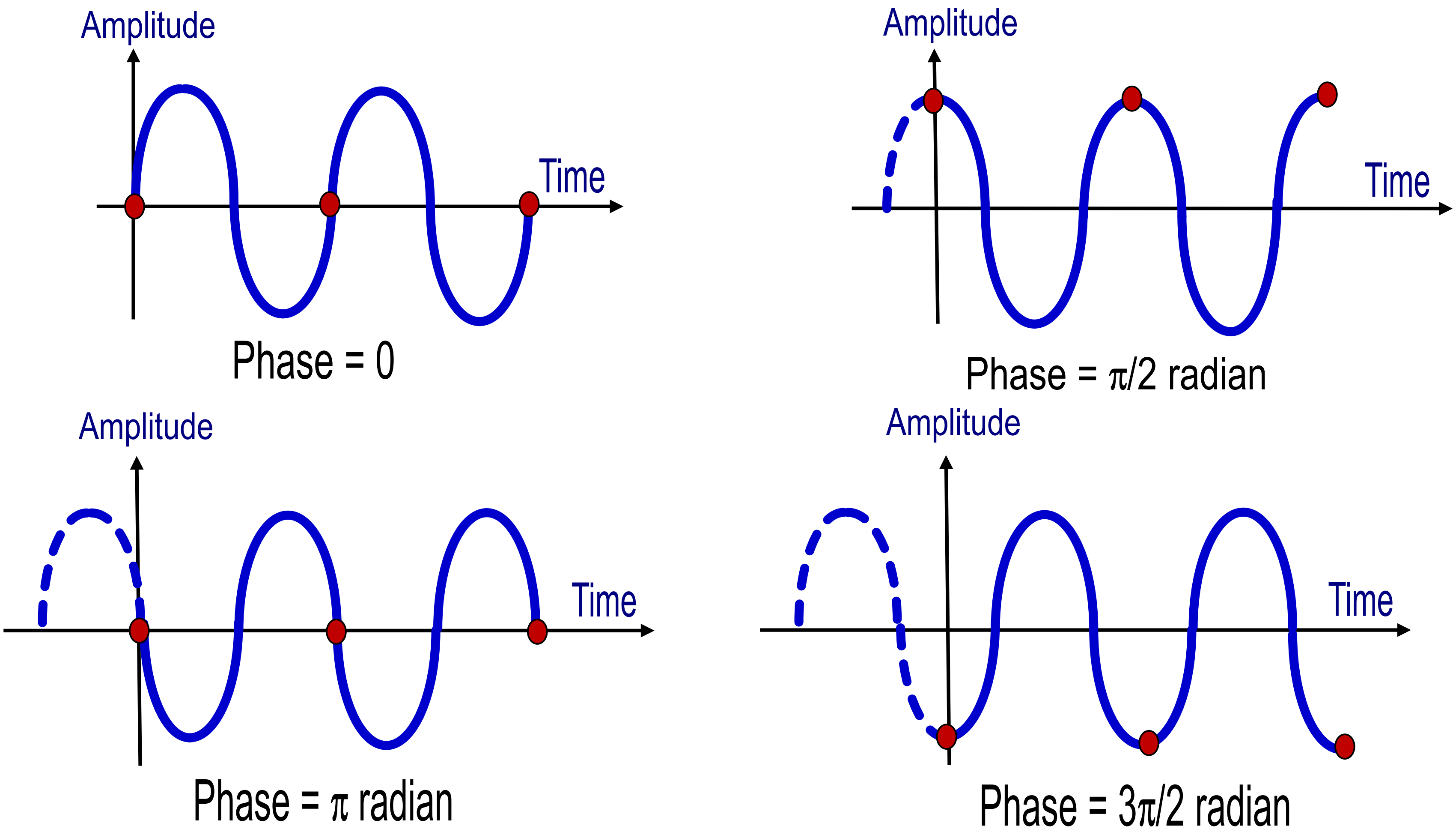
Carrier

BPSK



Binary Phase Shift Keying

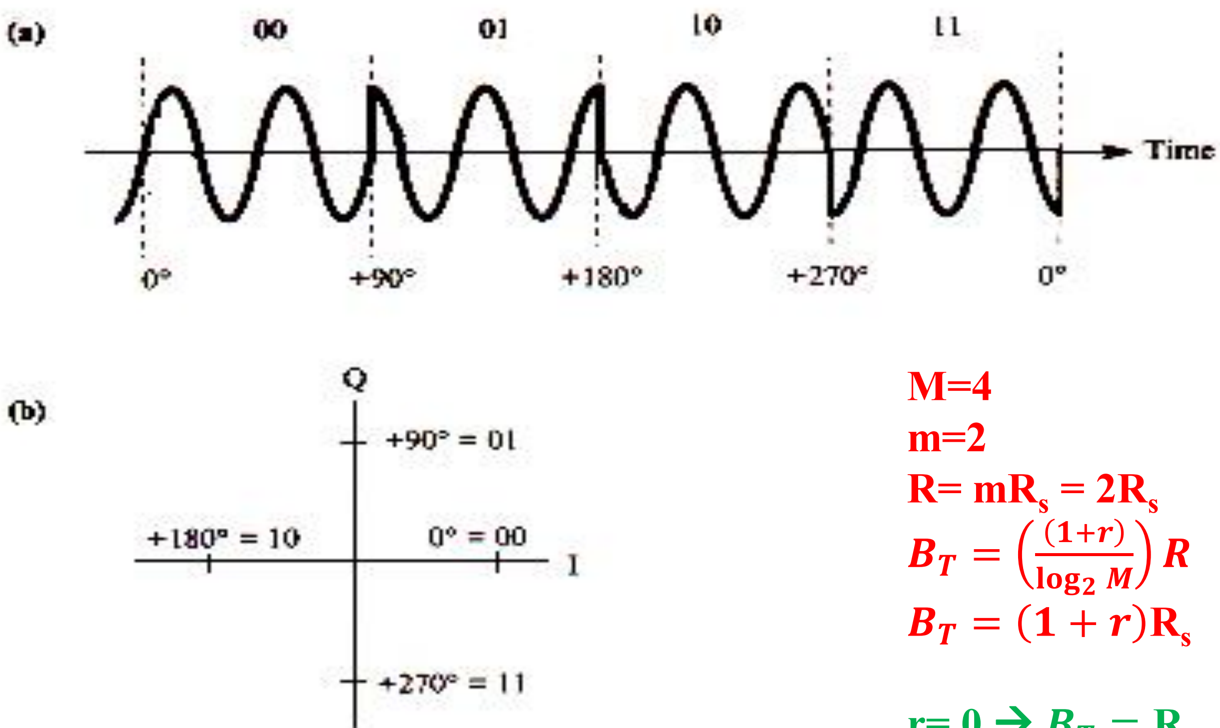
Different phases:



Multilevel Modulation

- More efficient use of bandwidth can be achieved if each signaling element represents more than one bit.
- Instead of a phase shift of 180^o, **Quadrature Phase-Shift Keying (QPSK)** or **(4-PSK)** technique uses phase shifts of multiple of 90^o.

4-PSK



4-PSK phase diagram

M=4

m=2

R= mR$_s$ = 2R$_s$

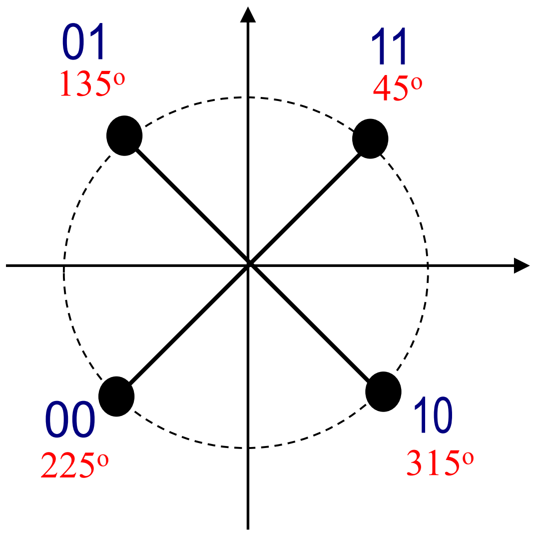
$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$

$B_T = (1 + r)R_s$

r= 0 → $B_T = R_s$

R= 2R$_s$

4-PSK



M=4

m=2

R= mR$_s$ = 2R$_s$

$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$

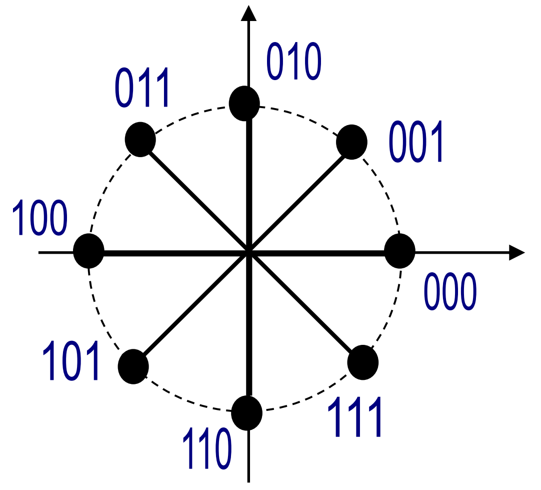
$B_T = (1 + r)R_s$

r= 0 → $B_T = R_s$

R= 2R$_s$

8-PSK

Tribit	Phase
000	0
001	45
010	90
011	135
100	180
101	225
110	270
111	315



8-PSK phase diagram

M=8
m=3
R= mR$_s$ = 3R$_s$

$$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$$

$$B_T = (1+r)R_s$$

r= 0 → $B_T = R_s$
R= 3R$_s$

Multilevel Phase Shift Keying

$$\text{Bit Rate (R)} = m \times R_s$$

Example:

Given a baud rate of 5000 baud for an **8-PSK** signal, what are the **bit rate**?

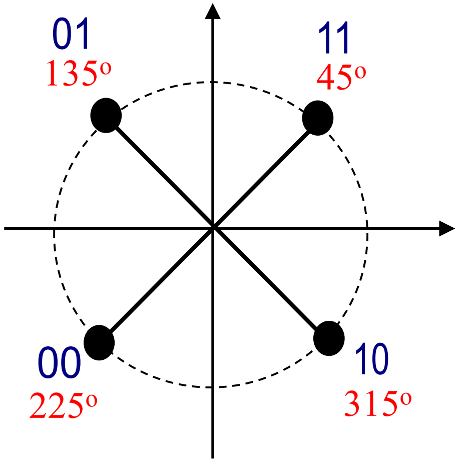
Solution:

$$R = m \times Rs = 3 \times 5000 = 15000 \text{ bps}$$

Quadrature PSK

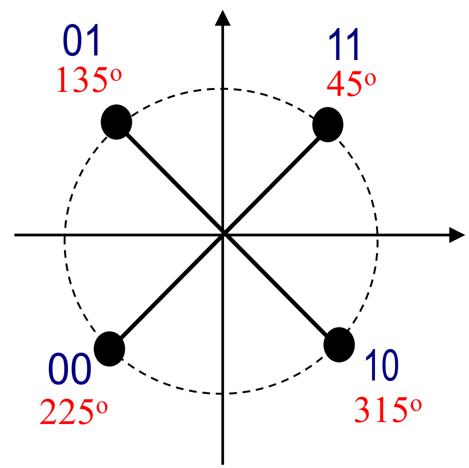
- get more efficient use if each signal element represents more than one bit
  - eg. shifts of $\pi/2$ (90^o)
  - each signal element represents two bits
  - More efficient use of bandwidth
  - split input data stream in two & modulate onto carrier & phase shifted carrier

$$
s(t) = \begin{cases} A\cos(2\pi f_c t + \frac{\pi}{4}) & 11 \\ A\cos(2\pi f_c t + 3\frac{\pi}{4}) & 01 \\ A\cos(2\pi f_c t - 3\frac{\pi}{4}) & 00 \\ A\cos(2\pi f_c t - \frac{\pi}{4}) & 10 \end{cases}
$$



Quadrature PSK

$$s(t) = \begin{cases} A\cos(2\pi f_c t + \frac{\pi}{4}) & 11 \\ A\cos(2\pi f_c t + 3\frac{\pi}{4}) & 01 \\ A\cos(2\pi f_c t - 3\frac{\pi}{4}) & 00 \\ A\cos(2\pi f_c t - \frac{\pi}{4}) & 10 \end{cases}$$



$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

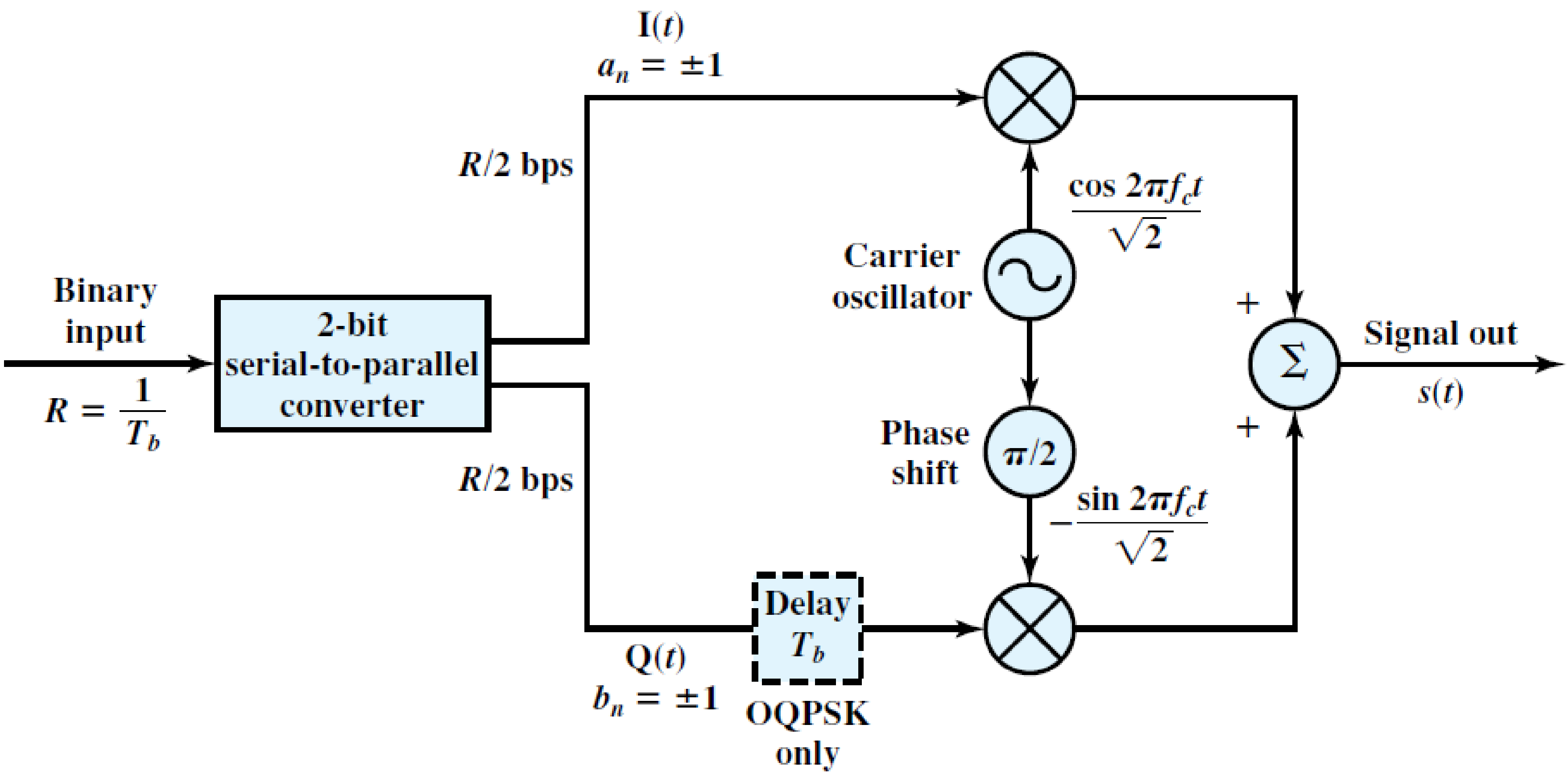
$$\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos(x \pm y) = \cos x \,.\, \cos y \mp \sin x \,.\, \sin y$$

$s(t)$	$I(t)$	$Q(t)$
$\cos(2\pi f_c t + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	+1	+1
$\cos(2\pi f_c t + \frac{3\pi}{4}) = \frac{-1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	-1	+1
$\cos(2\pi f_c t - \frac{3\pi}{4}) = \frac{-1}{\sqrt{2}}\cos(2\pi f_c t) + \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	-1	-1
$\cos(2\pi f_c t - \frac{\pi}{4}) = \frac{1}{\sqrt{2}}\cos(2\pi f_c t) + \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	+1	-1

$$\boldsymbol{s(t) = \frac{1}{\sqrt{2}} I(t) cos 2\pi f_c t - \frac{1}{\sqrt{2}} Q(t) sin 2\pi f_c t}$$

QPSK Modulators

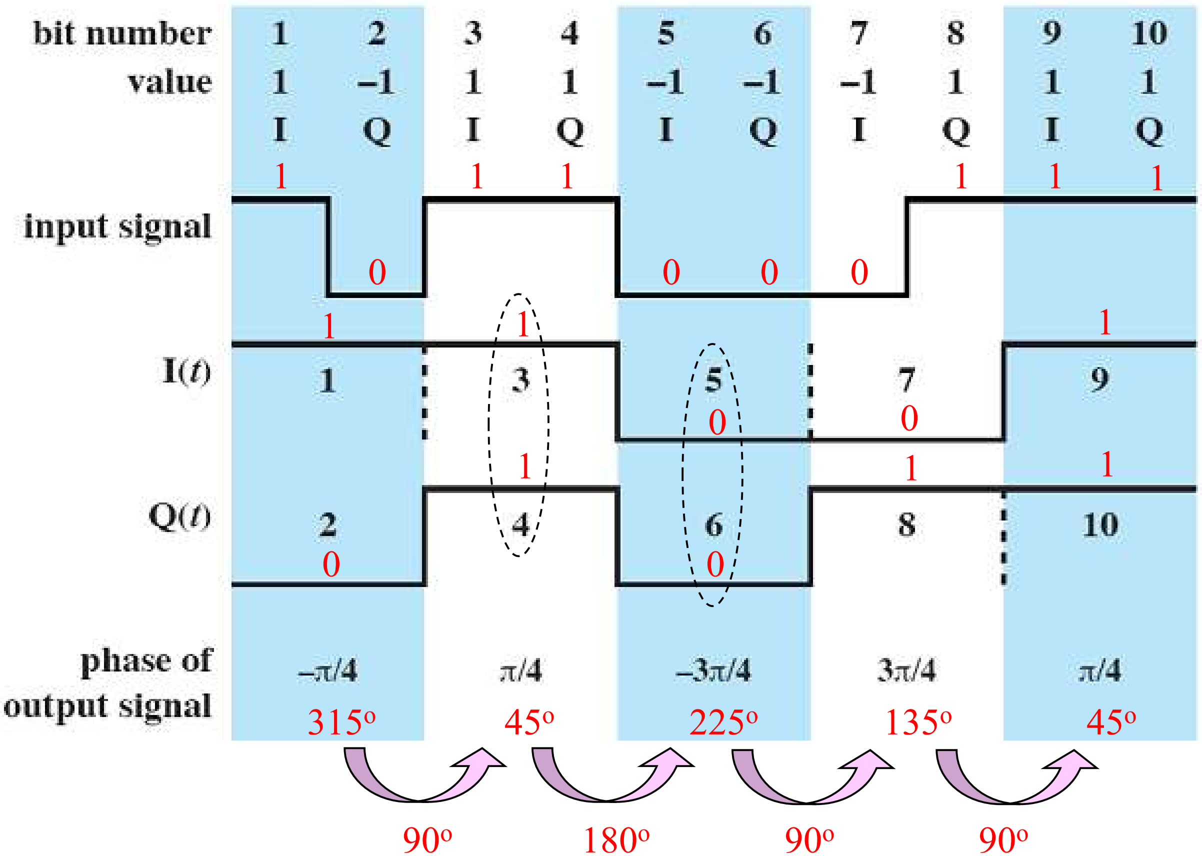


QPSK Modulation

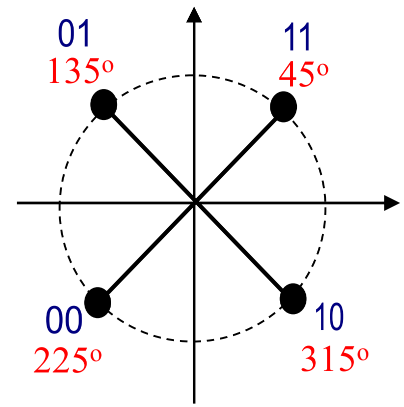
- Input is stream bin digits with rate **$R=1/T_b$**
- Converted to 2 bit streams of **R/2** bps each
  - taking alternate bits
  - I(t): in phase
  - Q(t): quadrature phase
- Carrier wave shifted by π/2 and used to modulate lower stream Q(t)
- Two modulated signals are then added
- QPSK transmitted signal:

$$\boxed{s(t) = \frac{1}{\sqrt{2}} I(t) \cos 2\pi f_c t - \frac{1}{\sqrt{2}} Q(t) \sin 2\pi f_c t}$$

QPSK Modulation



$$s(t) = \begin{cases} A\cos(2\pi f_c t + \pi/4) & 11 \\ A\cos(2\pi f_c t + 3\pi/4) & 01 \\ A\cos(2\pi f_c t - 3\pi/4) & 00 \\ A\cos(2\pi f_c t - \pi/4) & 10 \end{cases}$$



QPSK Modulation

$$\cos(x \pm y) = \cos x \cos y \quad \sin x \sin y$$

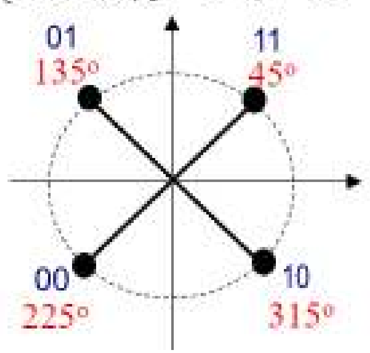
$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$s(t) = \begin{cases} A\cos(2\pi f_c t + \pi/4) & 11 \\ A\cos(2\pi f_c t + 3\pi/4) & 01 \\ A\cos(2\pi f_c t - 3\pi/4) & 00 \\ A\cos(2\pi f_c t - \pi/4) & 10 \end{cases}$$



$$\boxed{s(t) = \frac{1}{\sqrt{2}} I(t)\cos 2\pi f_c t - \frac{1}{\sqrt{2}} Q(t)\sin 2\pi f_c t}$$

$s(t)$	$I(t)$	$Q(t)$
$\cos(2\pi f_c t + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	+1	+1
$\cos(2\pi f_c t + \frac{3\pi}{4}) = \frac{-1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	-1	+1
$\cos(2\pi f_c t - \frac{3\pi}{4}) = \frac{-1}{\sqrt{2}}\cos(2\pi f_c t) + \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	-1	-1
$\cos(2\pi f_c t - \frac{\pi}{4}) = \frac{1}{\sqrt{2}}\cos(2\pi f_c t) + \frac{1}{\sqrt{2}}\sin(2\pi f_c t)$	+1	-1

OQPSK Modulators

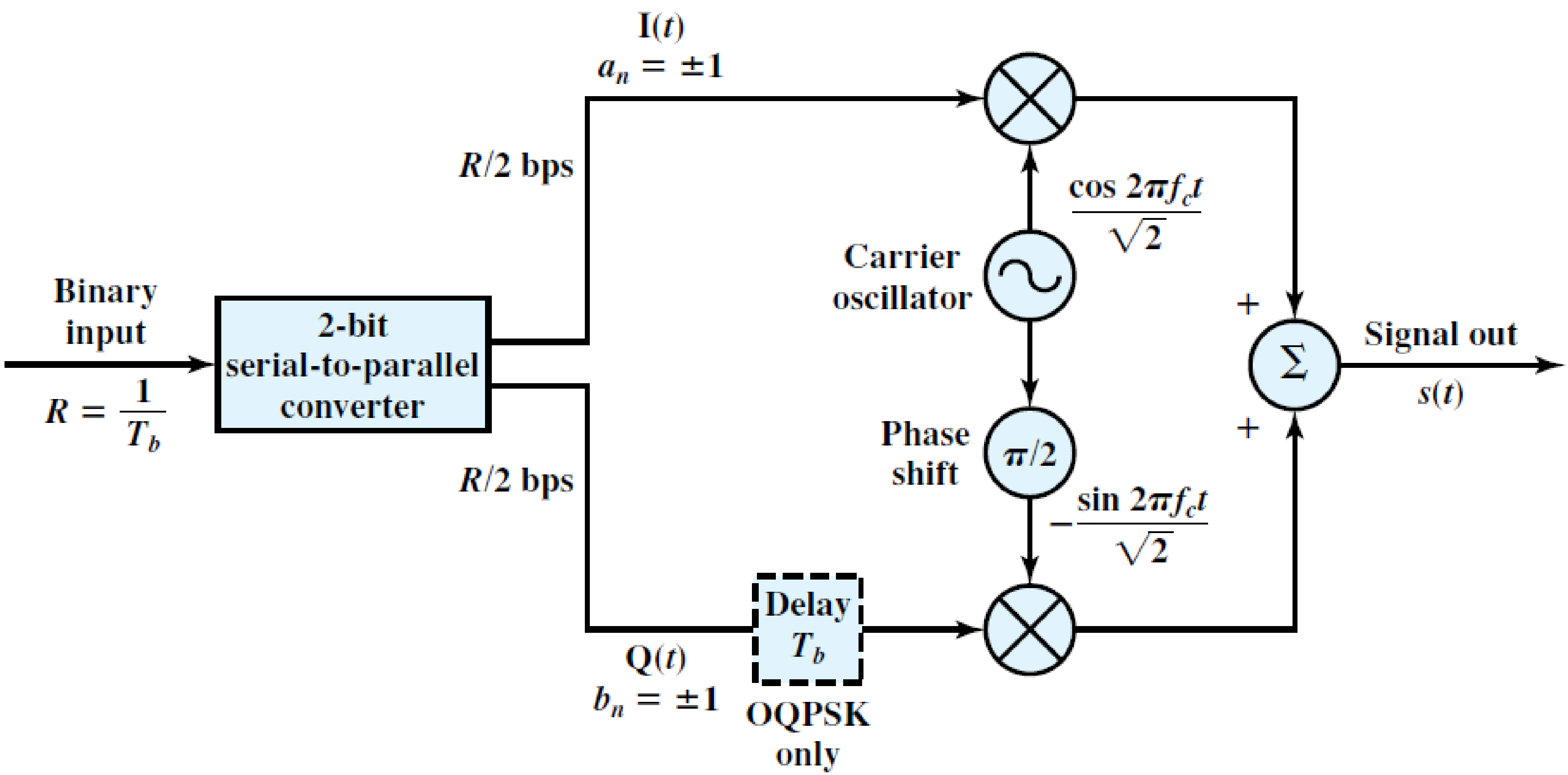
- Offset QPSK (OQPSK) or orthogonal PSK:

$$s(t) = \frac{1}{\sqrt{2}} I(t) \cos 2\pi f_c t - \frac{1}{\sqrt{2}} Q(t - T_b) \sin 2\pi f_c t$$

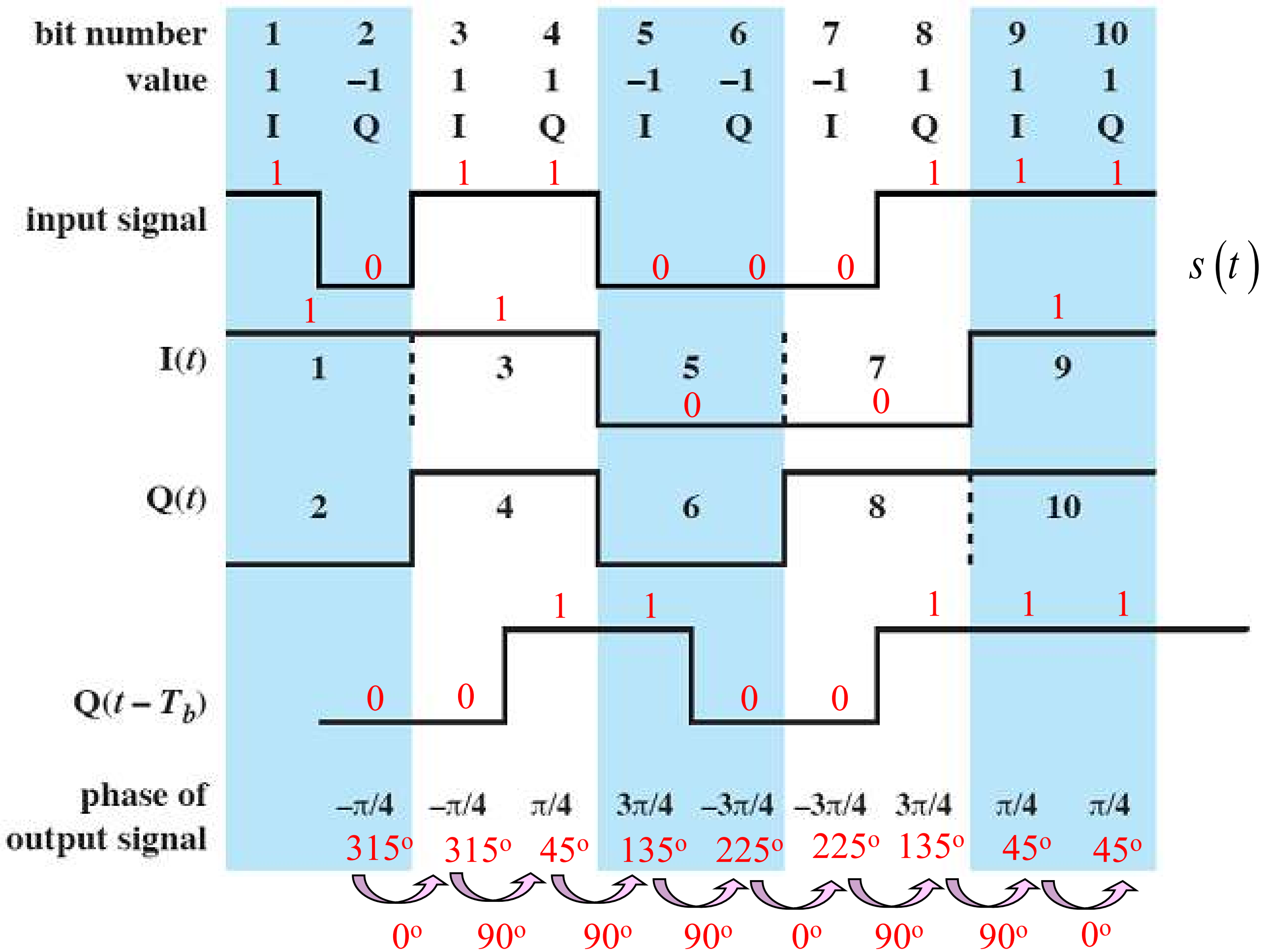
- In OQPSK, only one of the two bits in the pair can change at any time and thus the change in the combined signal never exceeds 90^o.

- Advantages:
  - **Physical limitation on phase modulators make large phase shifts at high transition rates difficult to implement.**
  - OQPSK provides superior performance when the transmission channel (including transmitter and receiver has significant nonlinear components. The effect of nonlinearities is a spreading of the signal bandwidth, which may result in adjacent channel interference. It is easier to control this spreading if the phase changes are smaller.

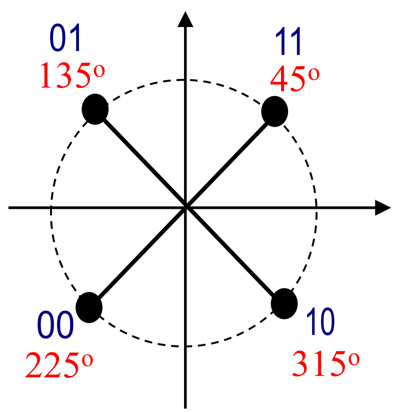
OQPSK Modulators



OQPSK Example



$$
s(t) = \begin{cases} A\cos(2\pi f_c + \pi/4) & 11 \\ A\cos(2\pi f_c + 3\pi/4) & 01 \\ A\cos(2\pi f_c - 3\pi/4) & 00 \\ A\cos(2\pi f_c - \pi/4) & 10 \end{cases}
$$



Performance

- ⌘ The transmission bandwidth B_T for ASK is of the form

$$\boxed{B_T = (1 + r)R}$$

where *$\boldsymbol{R}$ is the bit rate, and*
$\boldsymbol{r}$ is related to the technique by which the signal is filtered to establish a bandwidth for transmission; typically $0 < r < 1$.

- ⌘ Thus the bandwidth is directly related to the bit rate.
- ⌘ The preceding formula is also valid for PSK and, under certain assumptions, FSK.
- ⌘ With multilevel PSK (MPSK), significant improvements in bandwidth can be achieved.

Performance

- With **multilevel PSK (MPSK),** significant improvements in bandwidth can be achieved.

- MPSK:

$$\boxed{B_T = \left(\frac{1+r}{\log_2 M}\right)R}$$

Where $\log_2 M$ is the number of bits encoded per signal element, and
M is the number of different signal elements.

For (r=0) → $B_T = R / m$ **($B_T = R_s$ and $R = mR_s$)**

- For multilevel FSK (MFSK), we have

- MFSK

$$\boxed{B_T = \left(\frac{(1+r)M}{\log_2 M}\right)R}$$

- **For (r=0)** **→ $B_T = MR / m$** **→ $B_T = MR_s = B_T = 2Mf_d$**
- **For (r=0) and M=2** **→ $B_T = 2R / 1$** **→ $B_T = 2R$ and $R = R_s$**

Performance

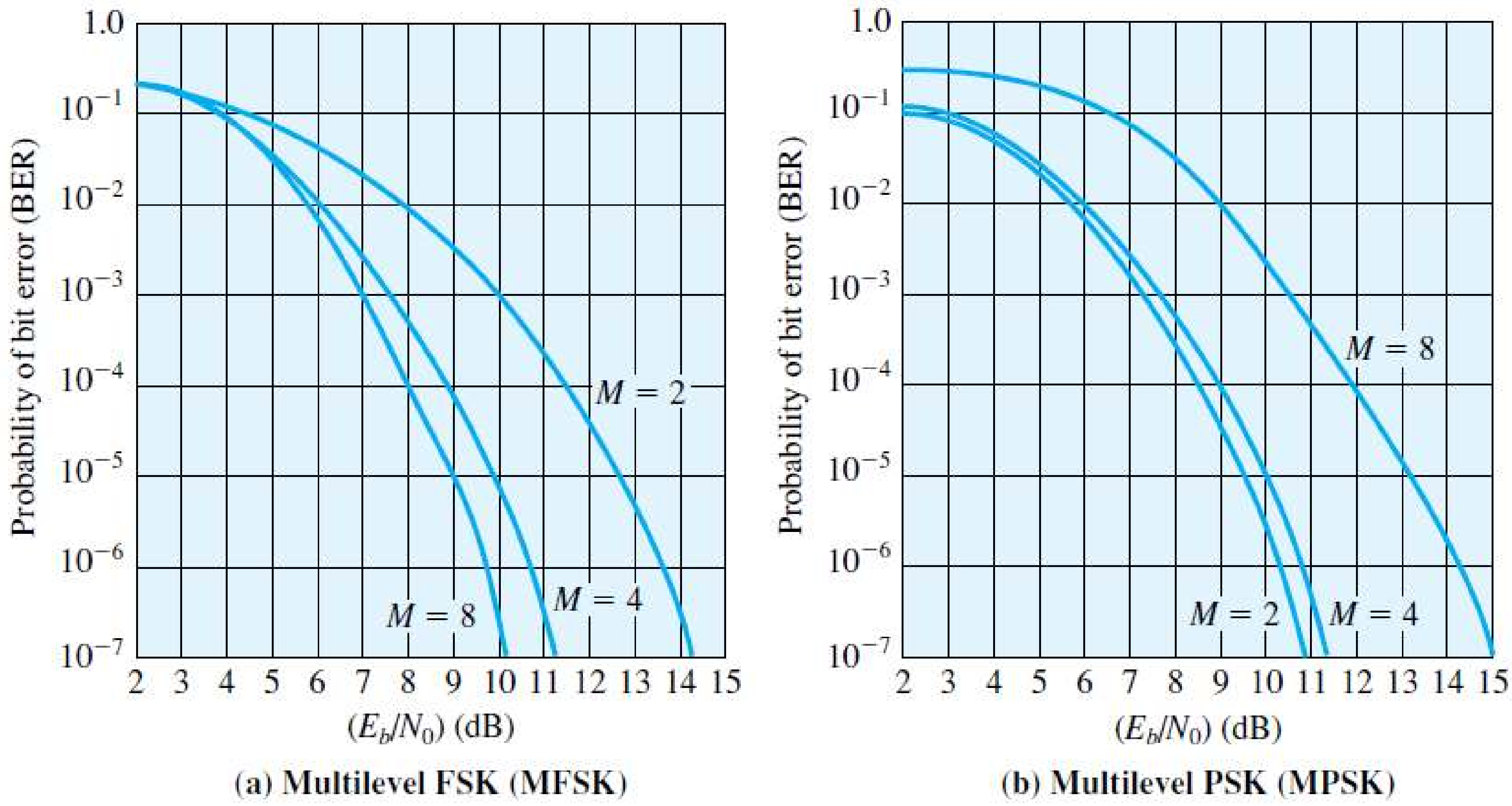
- **Bandwidth efficiency** parameter measures the efficiency with which bandwidth can be used to transmit data. The advantage of multilevel signaling methods now becomes clear.

Table 5.5 Bandwidth Efficiency (R/B_T) for Various Digital-to-Analog Encoding Schemes

	$r = 0$	$r = 0.5$	$r = 1$
ASK	1.0	0.67	0.5
FSK	0.5	0.33	0.25
Multilevel FSK			
$M = 4, L = 2$	0.5	0.33	0.25
$M = 8, L = 3$	0.375	0.25	0.1875
$M = 16, L = 4$	0.25	0.167	0.125
$M = 32, L = 5$	0.156	0.104	0.078
PSK	1.0	0.67	0.5
Multilevel PSK			
$M = 4, L = 2$	2.00	1.33	1.00
$M = 8, L = 3$	3.00	2.00	1.50
$M = 16, L = 4$	4.00	2.67	2.00
$M = 32, L = 5$	5.00	3.33	2.50

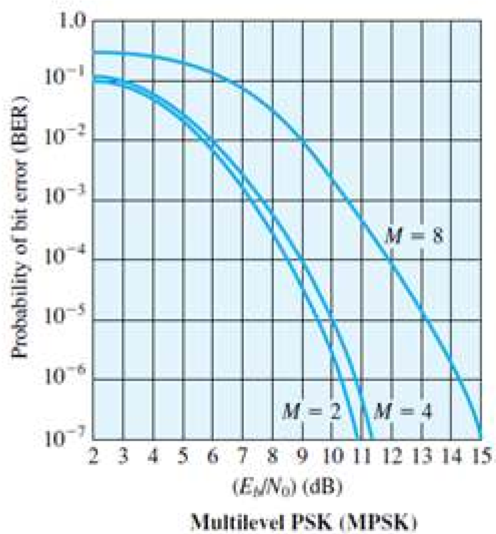
Performance

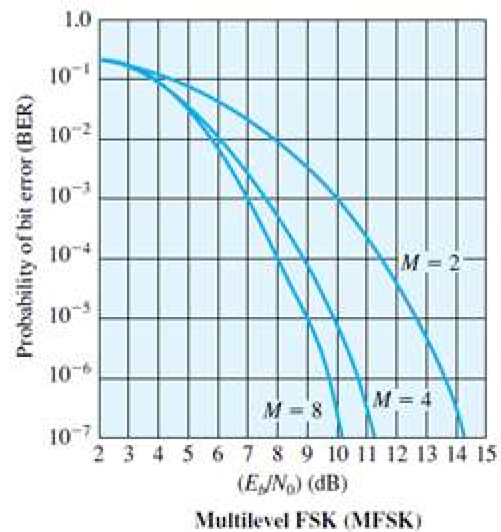
⌘ **MFSK & MPSK** have tradeoff between bandwidth efficiency and error performance



Performance

$$B_T = \left(\frac{1+r}{\log_2 M}\right)R$$





$$B_T = \left(\frac{(1+r)M}{\log_2 M}\right)R$$

$$\left(\frac{S}{N}\right)_{dB} = \left(\frac{E_b}{N_0}\right)_{dB} + \left(\frac{R}{B}\right)_{dB}$$

$$\left(\frac{E_b}{N_0}\right)_{dB} = \left(\frac{S}{N}\right)_{dB} - \left(\frac{R}{B}\right)_{dB}$$

In MPSK, QAM, and ASK

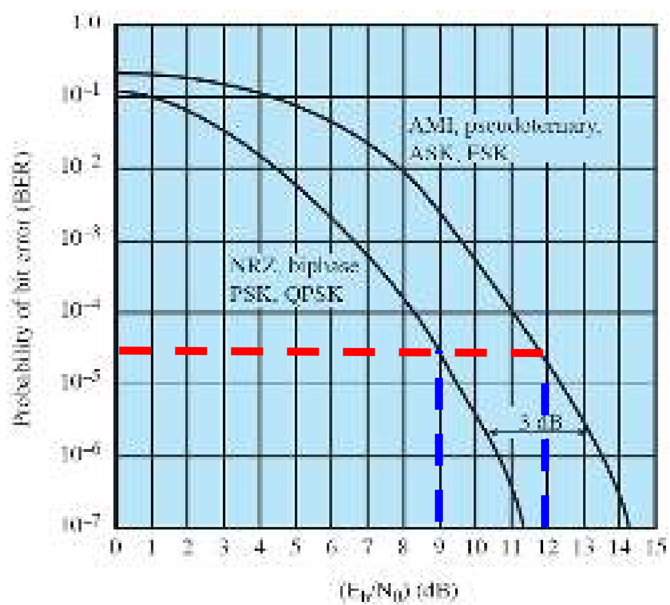
$$\eta = \frac{R}{B_T} = \frac{R}{\frac{1+r}{\log_2 M}R} = \frac{\log_2 M}{1+r} \quad \rightarrow \eta_{dB} = \left(\frac{R}{B_T}\right)_{dB} = 10\log_{10}\left(\frac{\log_2 M}{1+r}\right)$$

In MFSK,

$$\eta = \frac{R}{B_T} = \frac{R}{\frac{(1+r)M}{\log_2 M}R} = \frac{\log_2 M}{(1+r)M} \quad \rightarrow \eta_{dB} = \left(\frac{R}{B_T}\right)_{dB} = 10\log_{10}\left(\frac{\log_2 M}{(1+r)M}\right)$$

Performance of Digital to Analog Modulation Schemes

- **Bandwidth**
  - **ASK/PSK** bandwidth directly relates to bit rate
  - multilevel **PSK** gives significant improvements
- In the **presence of noise**:
  - bit error rate of **PSK** and **QPSK** are about 3dB superior to **ASK** and **FSK**
  - for **MFSK & MPSK** have tradeoff between bandwidth efficiency and error performance



Performance - Example

EXAMPLE 5.3 What is the bandwidth efficiency for FSK, ASK, PSK, and QPSK for a bit error rate of 10^{-7} on a channel with an SNR of 12 dB?

Using Equation (3.2), we have

$$\left(\frac{E_b}{N_0}\right)_{dB} = 12 \text{ dB} - \left(\frac{R}{B_T}\right)_{dB}$$

For FSK and ASK, from Figure 5.4,

$$\left(\frac{E_b}{N_0}\right)_{dB} = 14.2 \text{ dB}$$

$$\left(\frac{R}{B_T}\right)_{dB} = -2.2 \text{ dB}$$

$$\frac{R}{B_T} = 0.6$$

For PSK, from Figure 5.4,

$$\left(\frac{E_b}{N_0}\right)_{dB} = 11.2 \text{ dB}$$

$$\left(\frac{R}{B_T}\right)_{dB} = 0.8 \text{ dB}$$

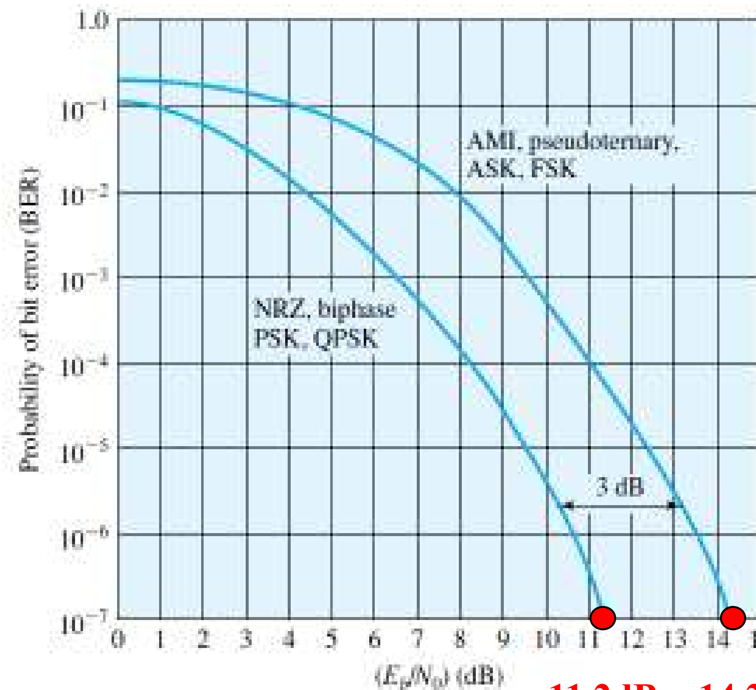
$$\frac{R}{B_T} = 1.2$$

R_s

The result for QPSK must take into account that the baud rate $D = R/2$. Thus

$$\frac{R}{B_T} = 2.4$$

$\eta = 2 \times 1.2 = 2.4$ bps/Hz

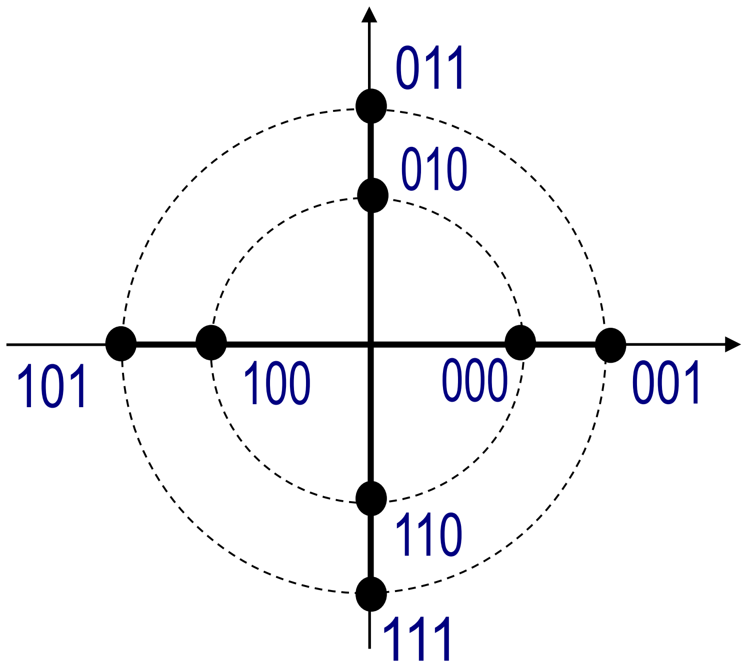


As the preceding example shows ASK and FSK exhibit the same bandwidth efficiency, PSK is better, and even greater improvement can be achieved with multilevel signaling.

Quadrature Amplitude Modulation (QAM)

- Higher bit rates are achieved using 8 and even 16 phase changes. In practice, however, there is a limit to how many phases can be used.
- Hence to increase the bit rate further, it is more common to introduce **amplitude** as well as **phase** variations of each vector. This type of modulation is then known as *Quadrature Amplitude Modulation* (**QAM**).
- 16-QAM has 16 levels per signal element, and hence 4-bit symbols.

8-QAM (2 amplitudes, 4 phases)



M=8

m=3

R= mR$_s$ = 3R$_s$

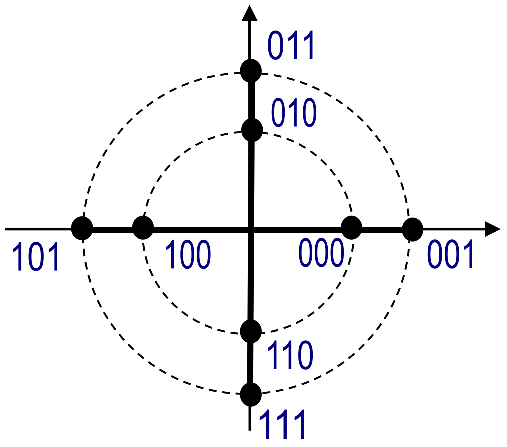
$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$

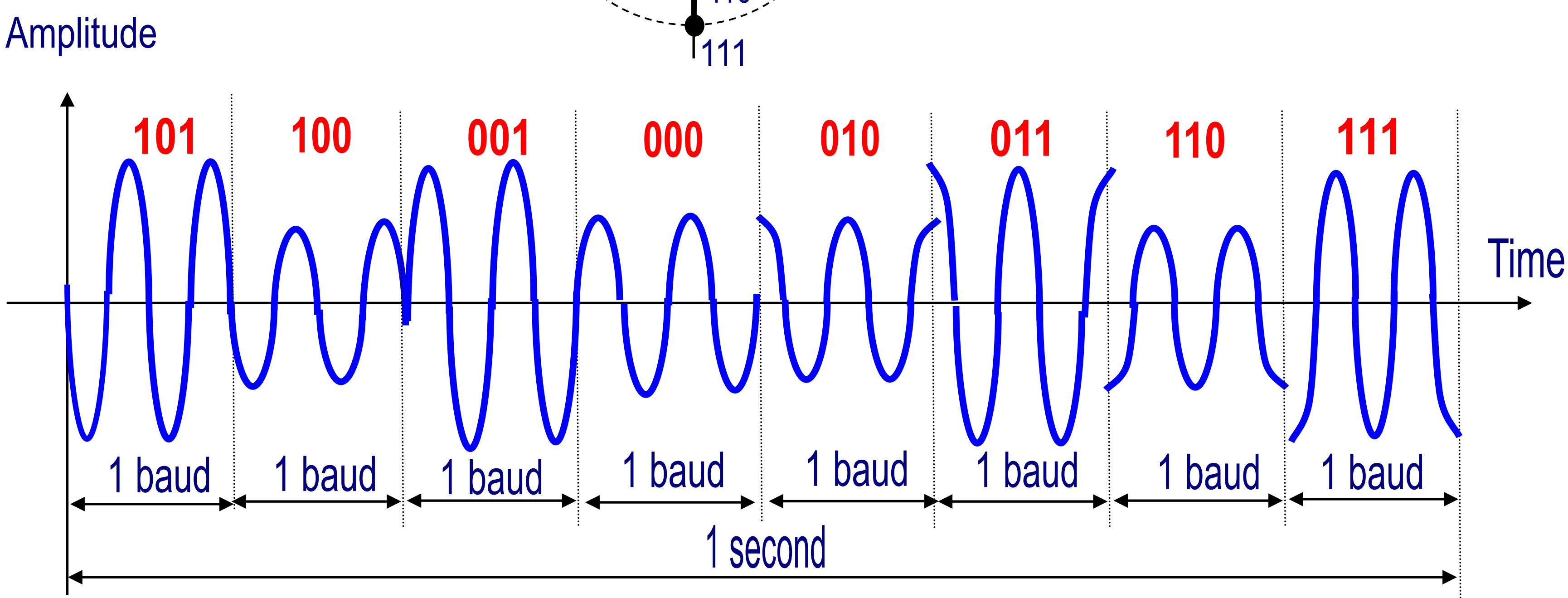
$B_T = (1+r)R_s$

r= 0 → $B_T = R_s$

R= 3R$_s$

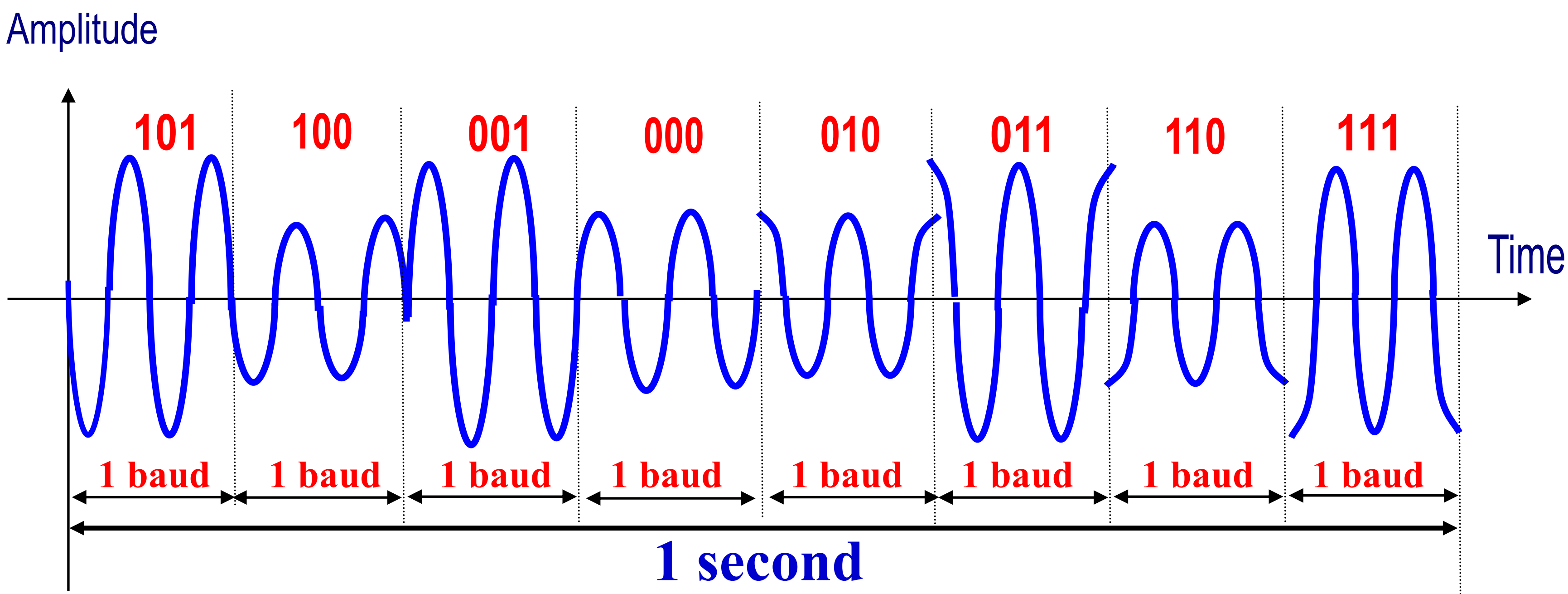
8-QAM – Example:





$$R = m \times R_s = 3 \times 8 = 24 \text{ bps}$$

2. Limited Bandwidth

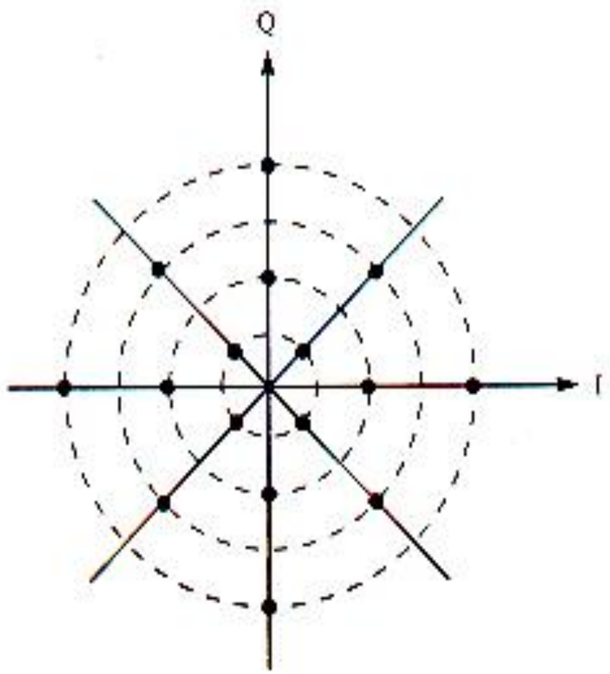


$R_s = 8$ baud $\rightarrow T_s = \frac{1}{R_s} = \frac{1}{8}$ second

$B = \frac{R_s}{2} = 4$ Hz

$R = m \times R_s = 3 \times 8 = 24$ bps $\rightarrow T_b = \frac{1}{R} = \frac{1}{24}$ second

16-QAM (4 amplitudes, 8 phases)



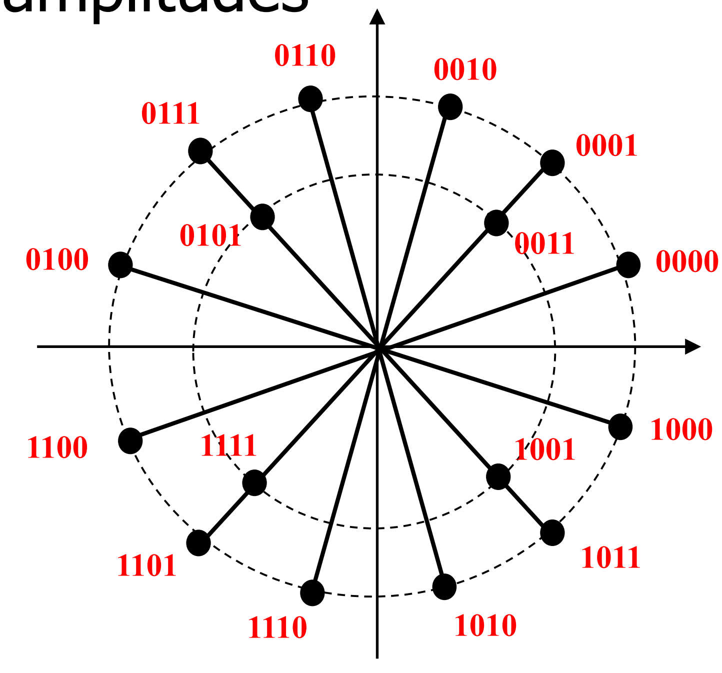
M=16
m=4
R= mR_s = 4R_s

$$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$$
$$B_T = (1 + r)R_s$$

r= 0 → $B_T = R_s$
R= 4R_s

16-QAM

- can use 8 phase angles & more than one amplitude
  - 9600bps modem uses 12 angles, four of which have two amplitudes



M=16
m=4
R= mR_s = 4R_s

$$B_T = \left(\frac{(1+r)}{\log_2 M}\right) R$$
$$B_T = (1+r)R_s$$

r= 0 → $B_T = R_s$
R= 4R_s

(QAM)

$$\text{Bit Rate (R)} = m \times R_s$$

- **Both amplitude and phase are changed**
- **Data rate is different from modulation rate**
- Example
  - M = 16 combinations of amplitude and phase
  - m = 4 bits/symbol
  - modulation rate $R_s = R/4$
  - If R_s = 2400 baud, R = 9600 bps
- High data rates over voice-grade lines

QAM – Examples:

(a) Compute the **baud rate (R_s)** for a 4800 bps **8-QAM** signal.

(b) Compute the **bit rate (R)** for a 1000 baud **16-QAM** signal.

(c) Compute the **baud rate (R_s)** for a 72000 bps **64-QAM** signal.

Solutions:

(a) $R_s = R \div m = 4800 \div 3 = 1600$ baud

(b) $R = R_s \times m = 1000 \times 4 = 4000$ bps

(c) $R_s = R \div m = 72000 \div 6 = 12000$ baud

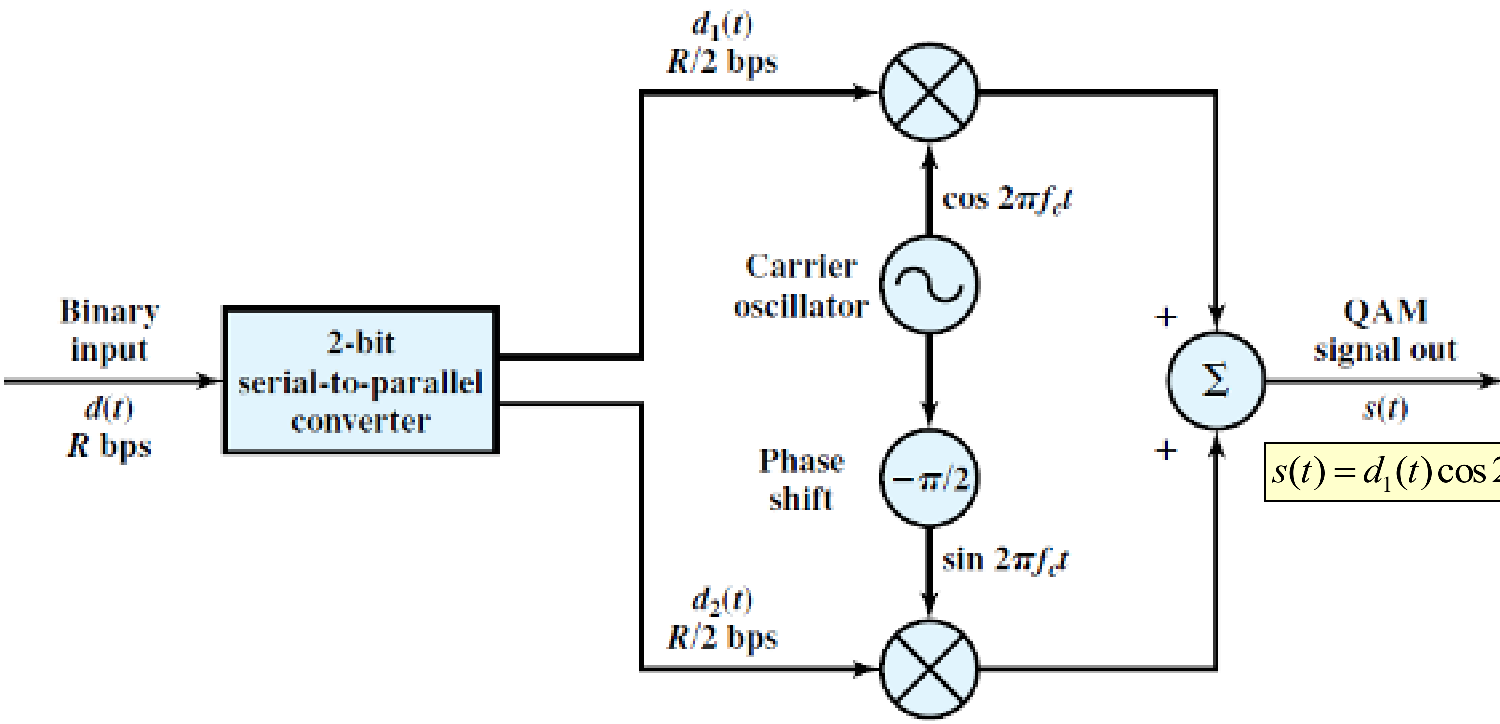
Bit and Baud Rate Comparison

Encoding	Units	Bits/Baud	Baud Rate	Bit Rate
ASK , FSK , 2-PSK	Bits	1	N	N
4-PSK , 4 QAM	Dibit	2	N	2N
8-PSK , 8-QAM	Tribit	3	N	3N
16-QAM	Quadbit	4	N	4N
32-QAM	Pentabit	5	N	5N
64-QAM	Hexabit	6	N	6N
128-QAM	Septabit	7	N	7N
256-QAM	Octabit	8	N	8N

QAM

- **QAM** used on **asymmetric digital subscriber line (ADSL)** and some wireless standards
- **QAM** is a combination of ASK and PSK
- send two different signals simultaneously on same carrier frequency
  - **use two copies of carrier, one shifted 90°**
  - **each carrier is ASK modulated**
  - **two independent signals over same medium**
  - **demodulate and combine for original binary output**

QAM Modulator



$$s(t) = d_1(t)\cos 2\pi f_c t + d_2(t)\sin 2\pi f_c t$$

QAM Variants

- **two level ASK**
  - each of two streams in one of two states
  - four state system
  - essentially QPSK
- **four level ASK**
  - combined stream in one of 16 (2^4) states
- systems using **64 and 256 state** have been implemented.
- improved data rate for given bandwidth
  - but increased potential error rate

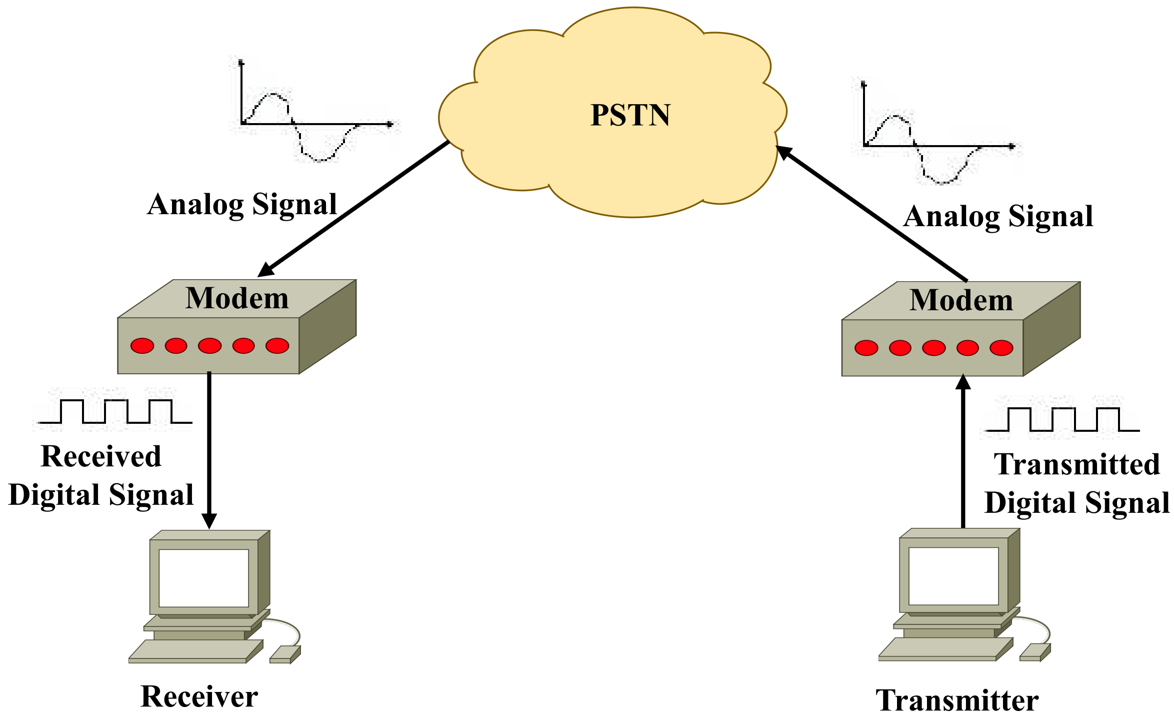
Modem

- A ***modem*** converts the digital signal generated by the computer into an analog signal to be carried by a public phone line. It is also converts the analog signals receiver over a phone line into digital signals usable by the computer.

- The term *modem* is composite word that refers to a signal *modulator* and a signal *demodulator*.

- A *modulator* treats a digital signal as a series of 1s and 0s, and so can transform it into an analog signal by using the digital-to-analog mechanisms of **ASK**, **FSK**, **PSK**, and **QAM**.

Modem


PSTN
Analog Signal
Analog Signal
Modem
Modem
Received
Digital Signal
Transmitted
Digital Signal
Receiver
Transmitter

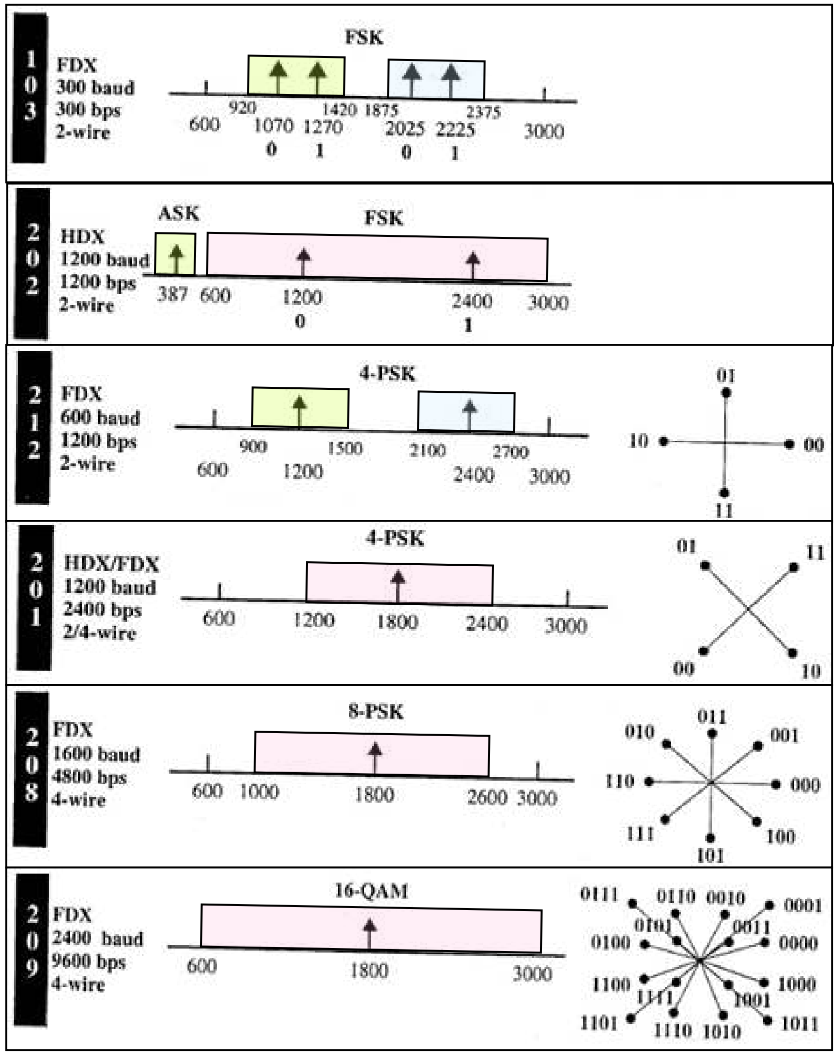
Modem

Modem Speeds:

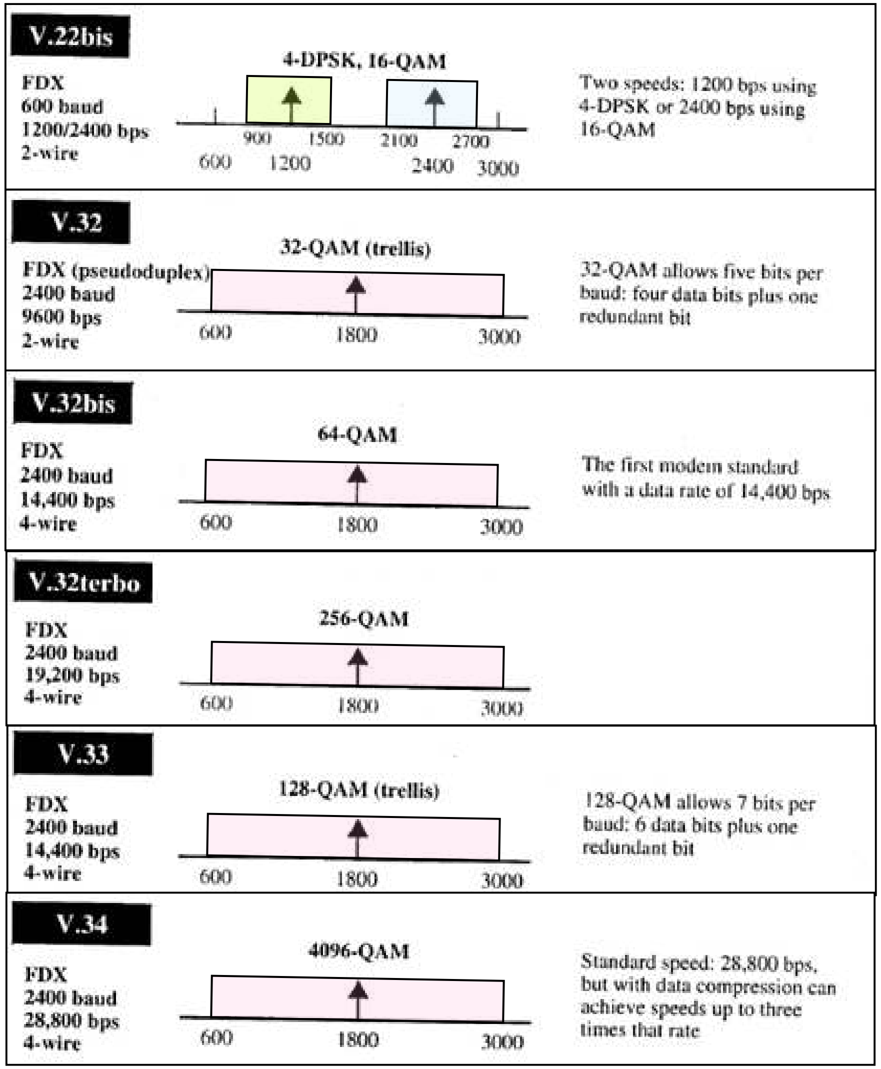
Theoretical Bit Rates for Modems:

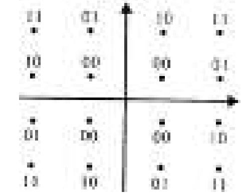
Encoding	Half-Duplex	Full-Duplex
ASK , FSK , 2-PSK	2400	1200
4-PSK , 4 QAM	4800	2400
8-PSK , 8-QAM	7200	3600
16-QAM	9600	4800
32-QAM	12000	6000
64-QAM	14400	7200
128-QAM	16800	8400
256-QAM	19200	9600

Modems Standards

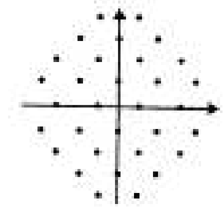


Modems Standards

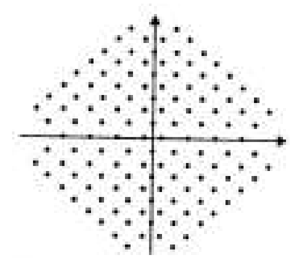




v.22bis



v.32



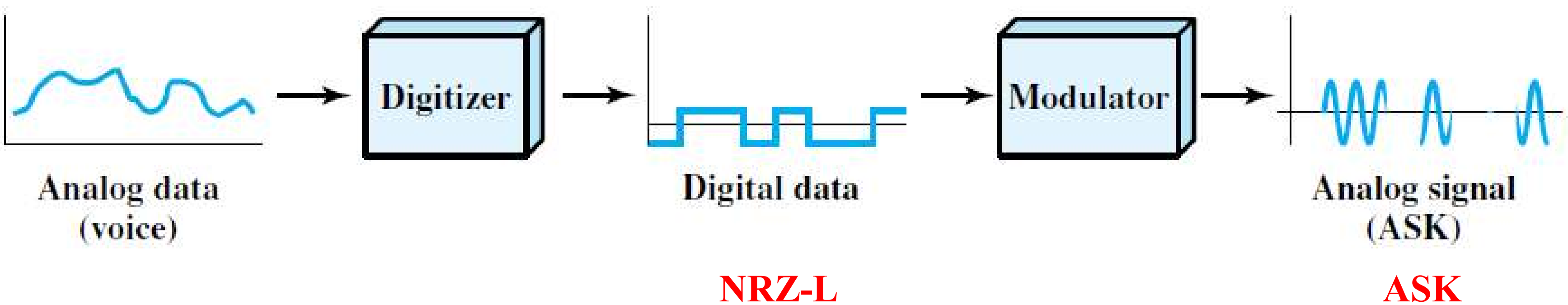
v.33

Encoding Techniques

Analog Data, Digital Signal

Analog Data, Digital Signal

- **Digitization** is **conversion of analog data into digital data** which can then:
  - be transmitted using NRZ-L
  - be transmitted using code other than NRZ-L
  - be converted to analog signal



Analog Data, Digital Signal

- **Why convert later to analog signal ?**
  - transmission media requirements
    - **microwave** only transmit analog signal
- **Codec (coder-decoder)**
  - **device** used for converting analog data to digital
  - also recover analog data from digital signal
- Analog to digital conversion done using a **codec**
  - **Pulse Code Modulation (PCM)**
  - **Delta Modulation**

Sampling Theorem

SAMPLING THEOREM: If a signal $f(t)$ is sampled at regular intervals of time and at a rate higher than twice the highest signal frequency, then the samples contain all the information of the original signal. The function $f(t)$ may be reconstructed from these samples by the use of a lowpass filter.

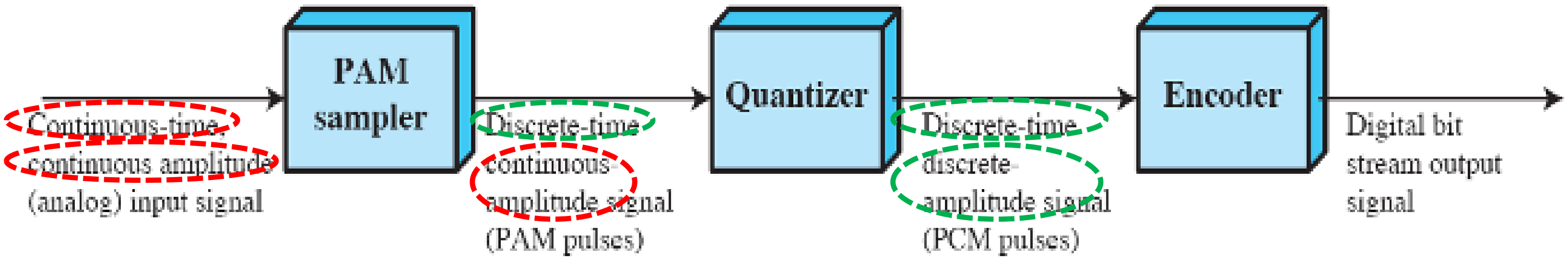
Pulse Code Modulation (PCM)

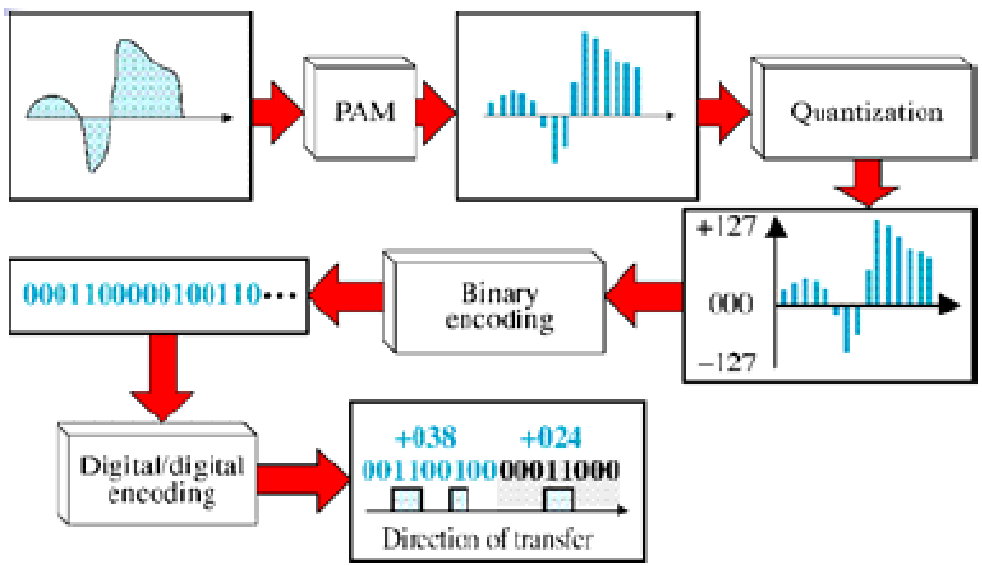
- **Nyquest Sampling Theorem:**
  - "**If a signal *f*(*t*) is sampled at regular intervals at a rate higher than twice the highest signal frequency, the samples contain all information in original signal**"
  - eg. 4000Hz voice data, requires 8000 sample per sec
- Strictly have analog samples
  - The sampled signal is first **converted into a pulse stream**, **the amplitude of each pulse being equal to the amplitude of the original analog signal at the sampling instant.** The resulting signal is known as a **Pulse Amplitude Modulated (PAM)** signal.
- Voice data limited to below **4000Hz**
- Require **8000 sample per second**
- Analog samples (Pulse Amplitude Modulation, PAM)
- Each sample assigned digital value

Pulse Code Modulation (PCM)

- **Quantized**
  - **Quantizing error or noise**
  - Approximations mean it is impossible to recover original exactly
- **8 bit sample gives 256 levels**
- Quality comparable with analog transmission
- **8000 samples per second of 8 bits each gives 64kbps**

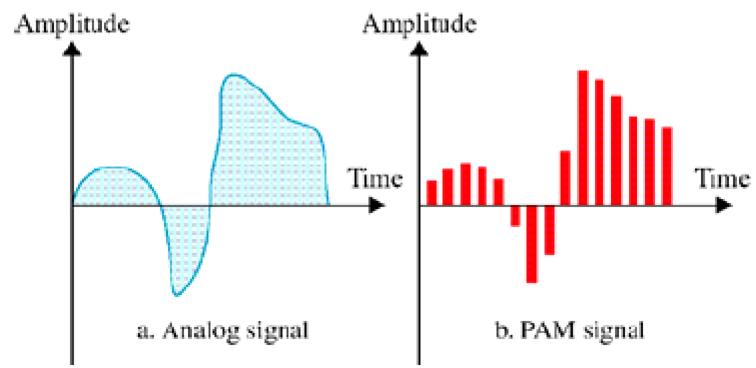
From Analog to PCM



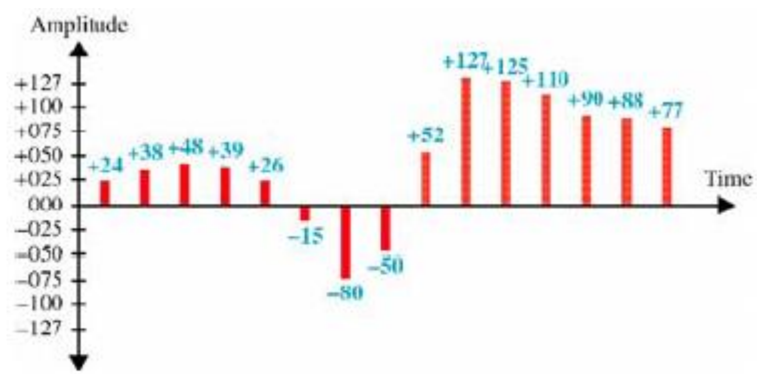


From Analog to PCM

Step 1: Pulse Amplitude Modulation (PAM)



Step 2: Quantized PAM Signal

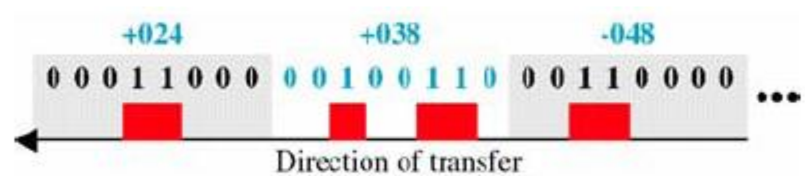


Step 3: Quantizing using Sign & Magnitude

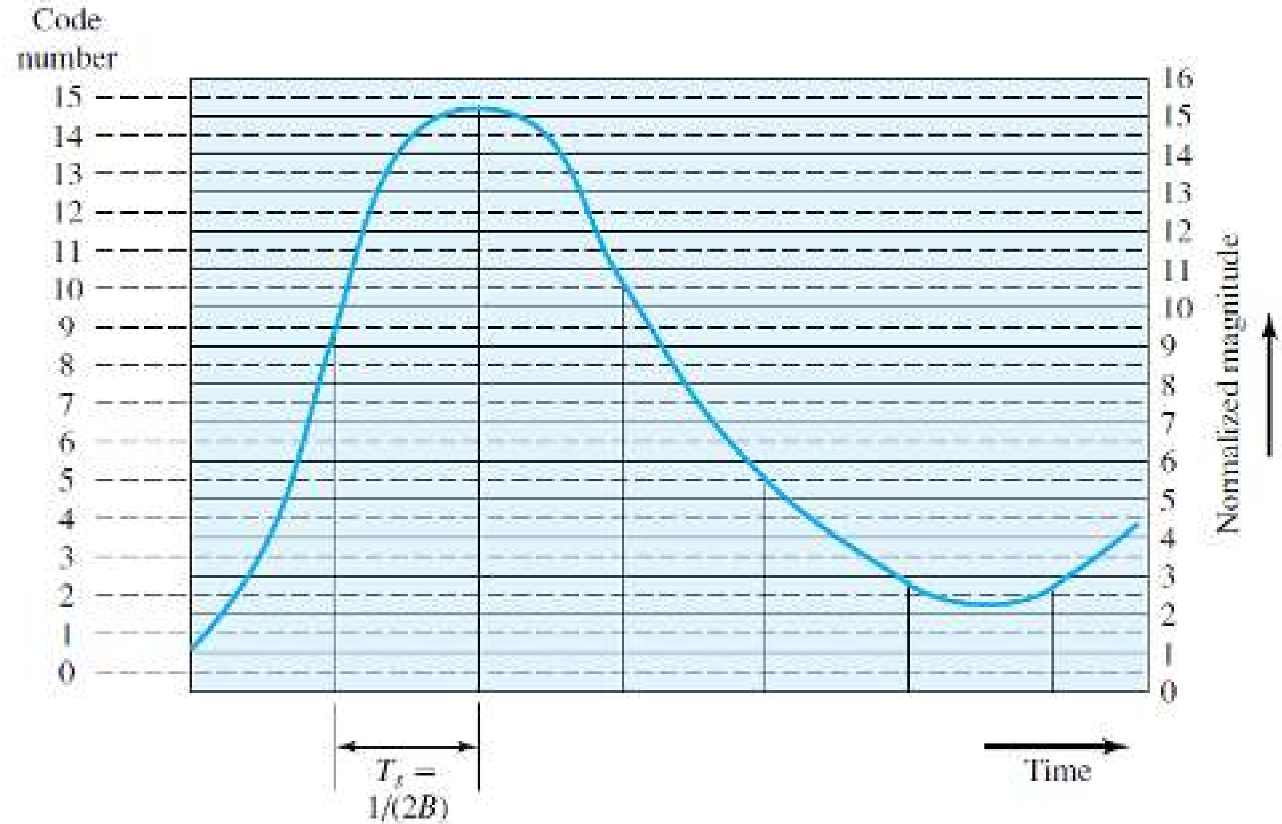
+024	00011000	−015	10001111	+125	01111101
+038	00100110	−080	11010000	+110	01101110
+048	00110000	−050	10110010	+090	01011010
+039	00100111	+052	00110110	+088	01011000
+026	00011010	+127	01111111	+077	01001101

Sign bit
+ is 0 − is 1

Step 4: Pulse Code Modulation (PCM)

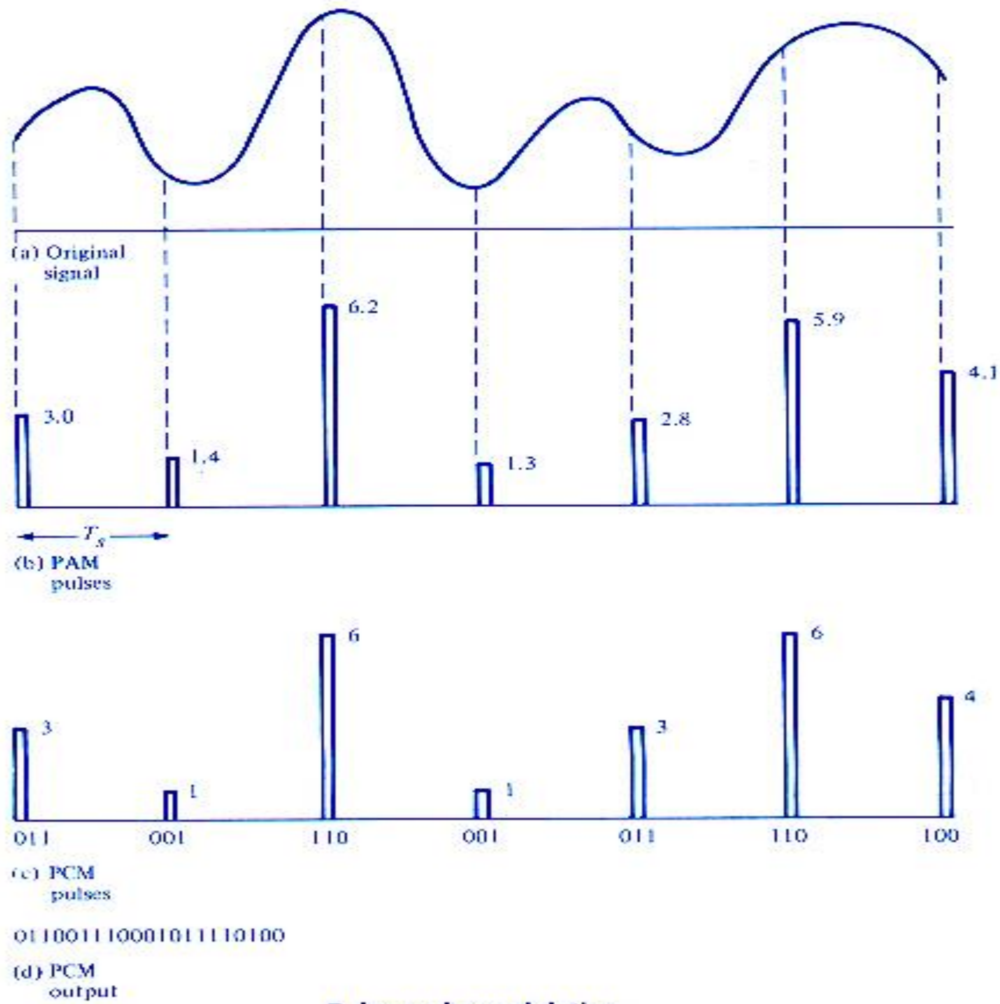


PCM



PAM value	1.1	9.2	15.2	10.8	5.6	2.8	2.7
Quantized code number	1	9	15	10	5	2	2
PCM code	0001	1001	1111	1010	0101	0010	0010

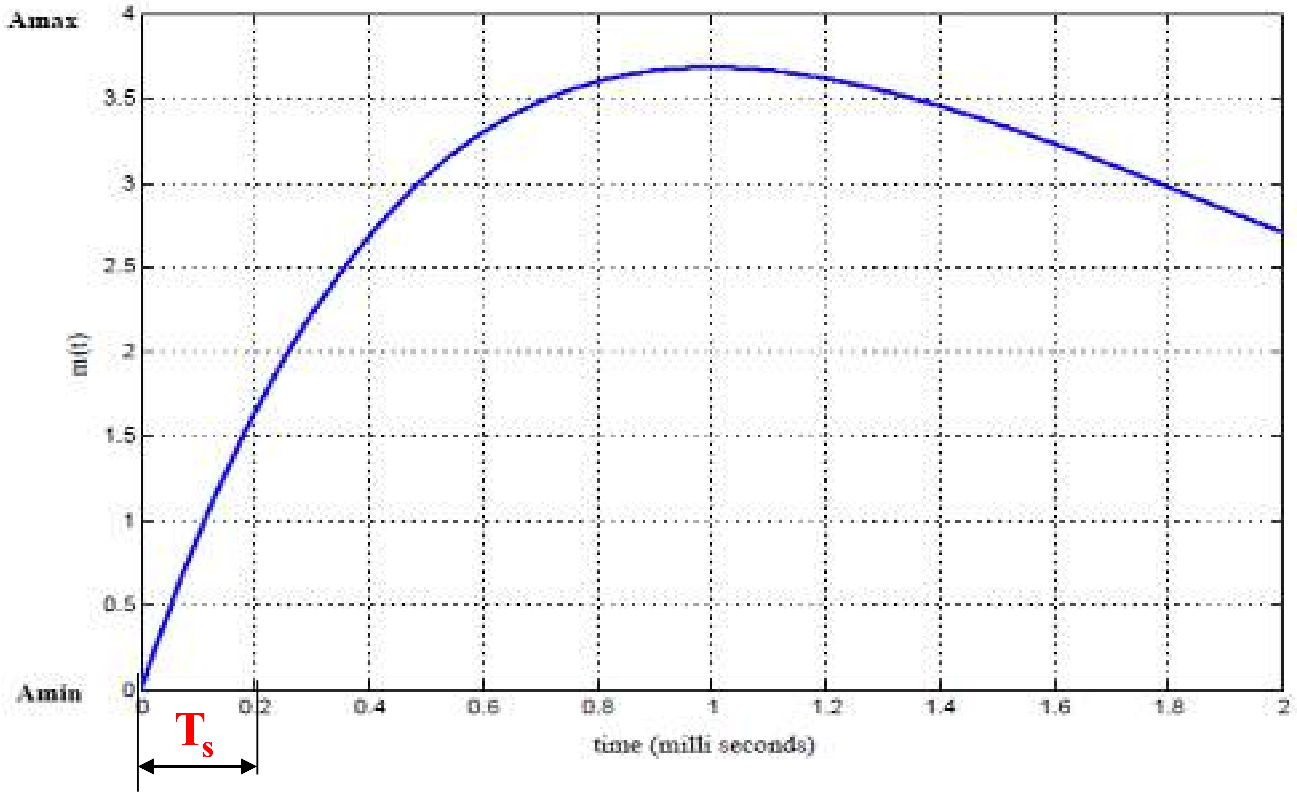
PCM



Pulse-code modulation.

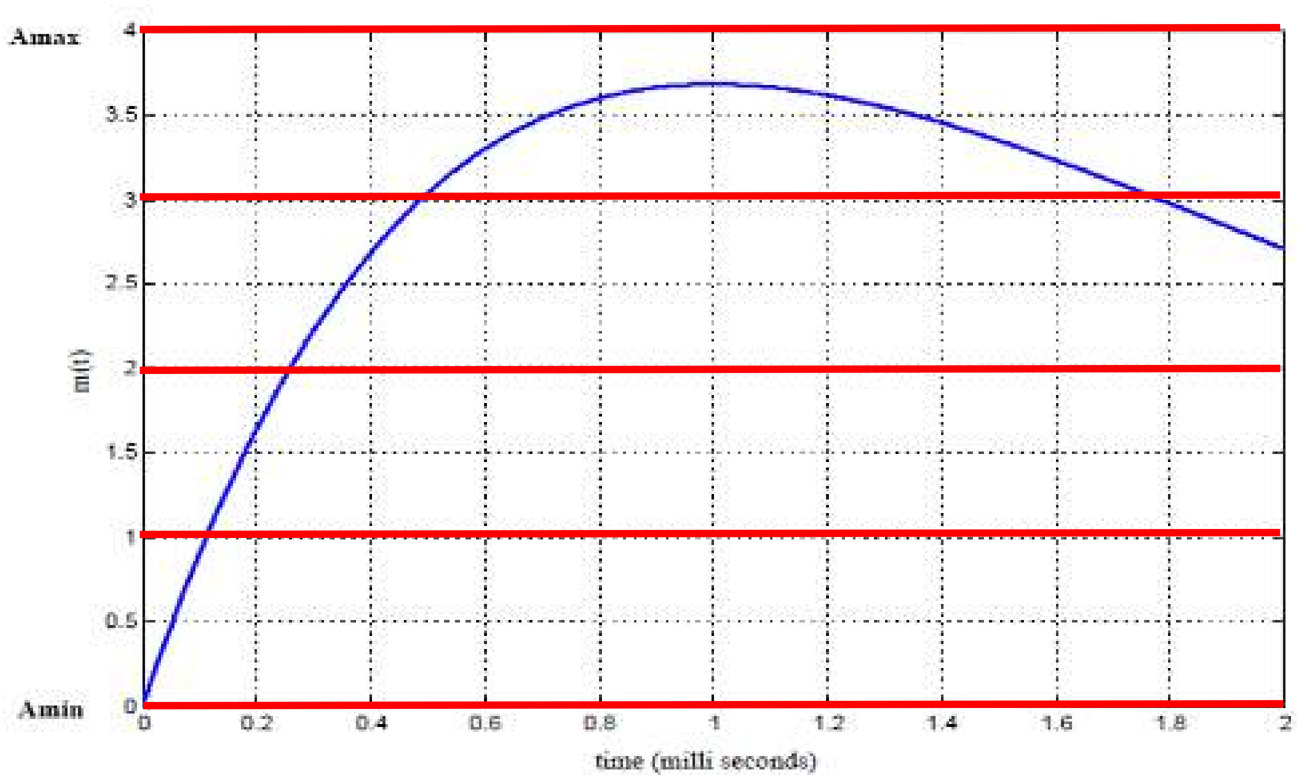
PCM – Example 1

Write the output PCM sequence using 4 levels.



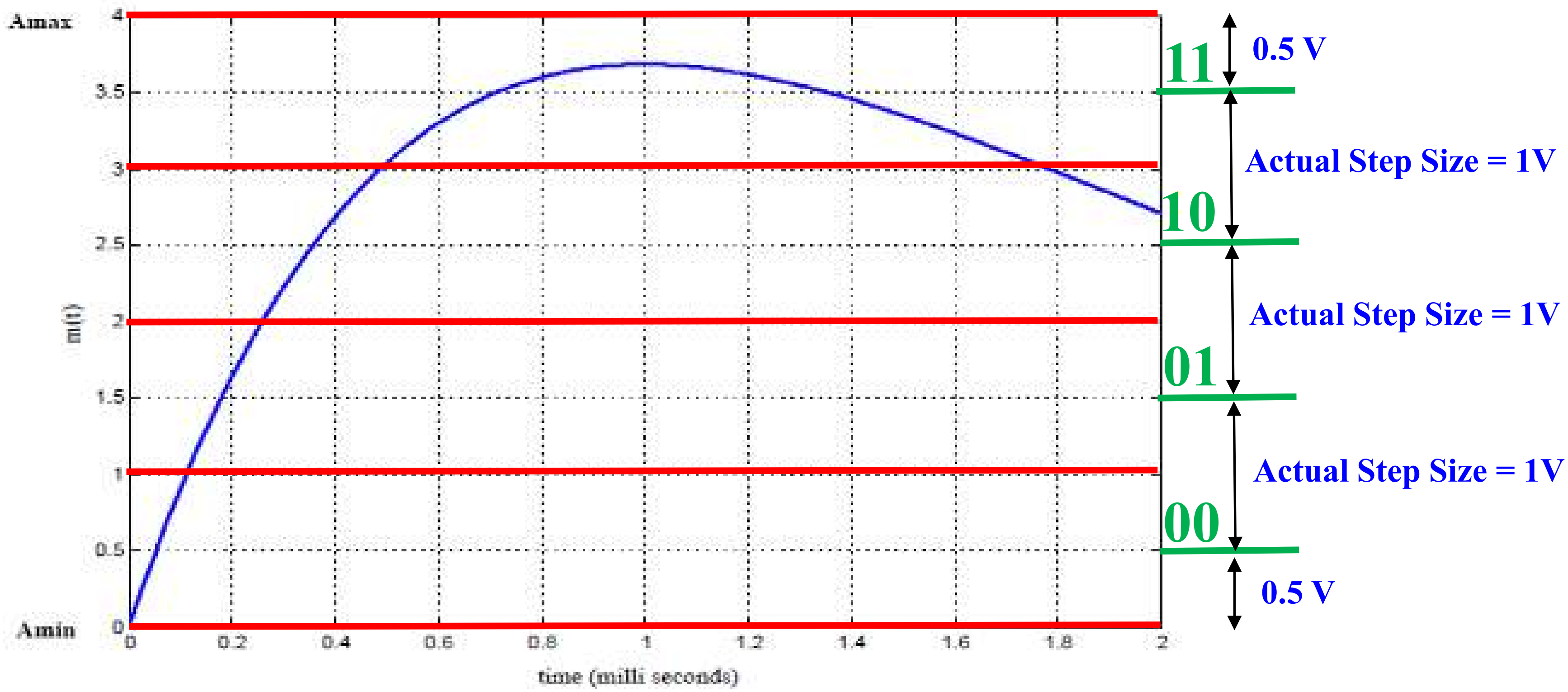
Sampling Rate = $2 \times f_{max}$ → Sampling Rate= $\frac{1}{T_s}$ → Sampling Rate= $\frac{1}{0.0002}$= 5000 Samples/sec

PCM – Example 1



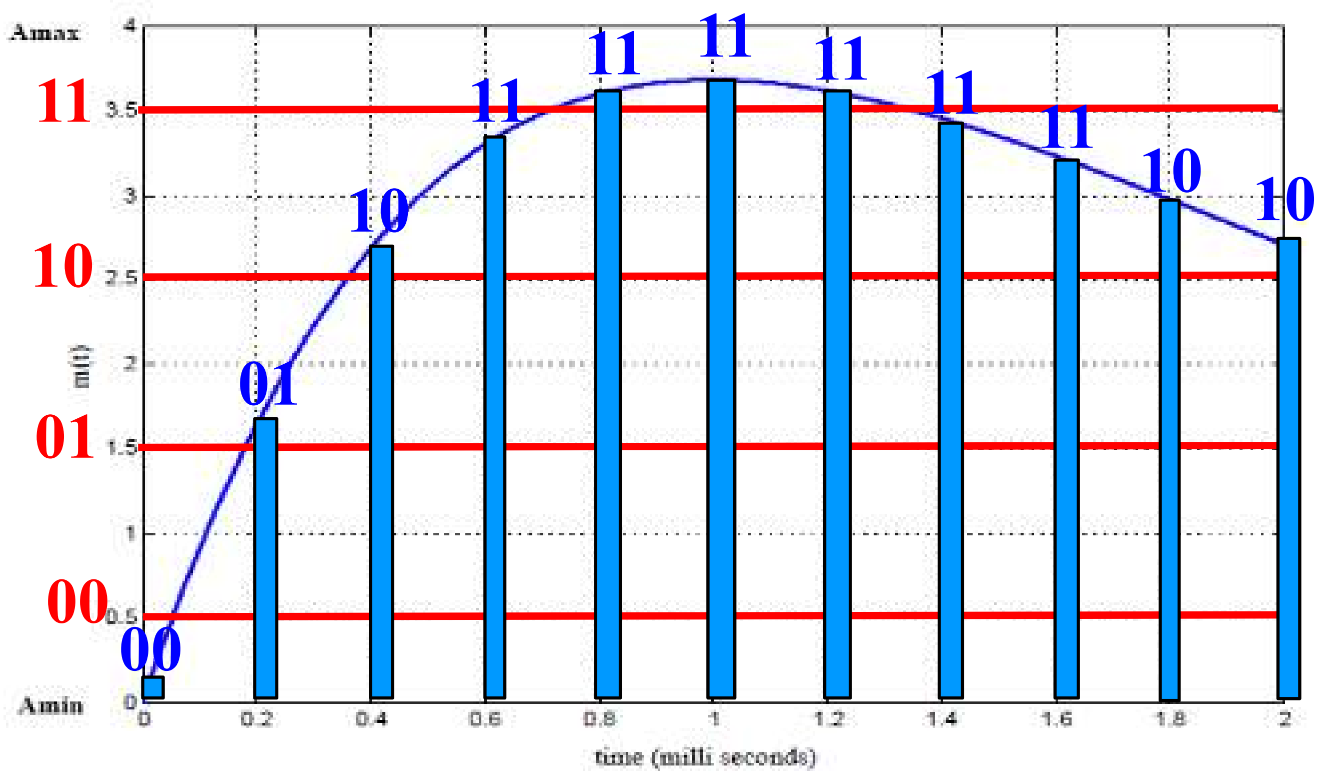
5 Levels

PCM – Example 1



$$\textbf{Actual Resolution} = \pm\frac{\textit{Actual Step Size}}{2} = \pm 0.5\ V$$

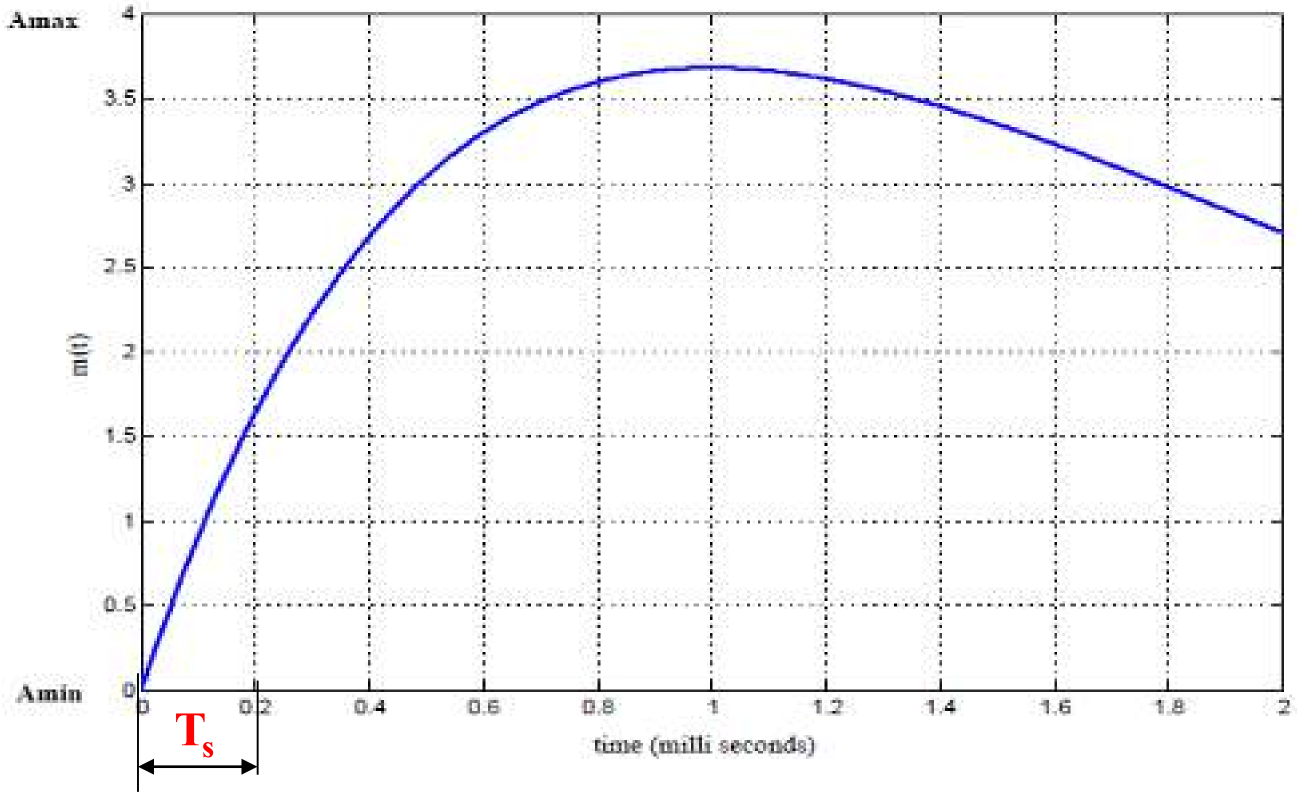
PCM – Example 1



Output PCM sequence = 00 01 10 11 11 11 11 11 11 10 10

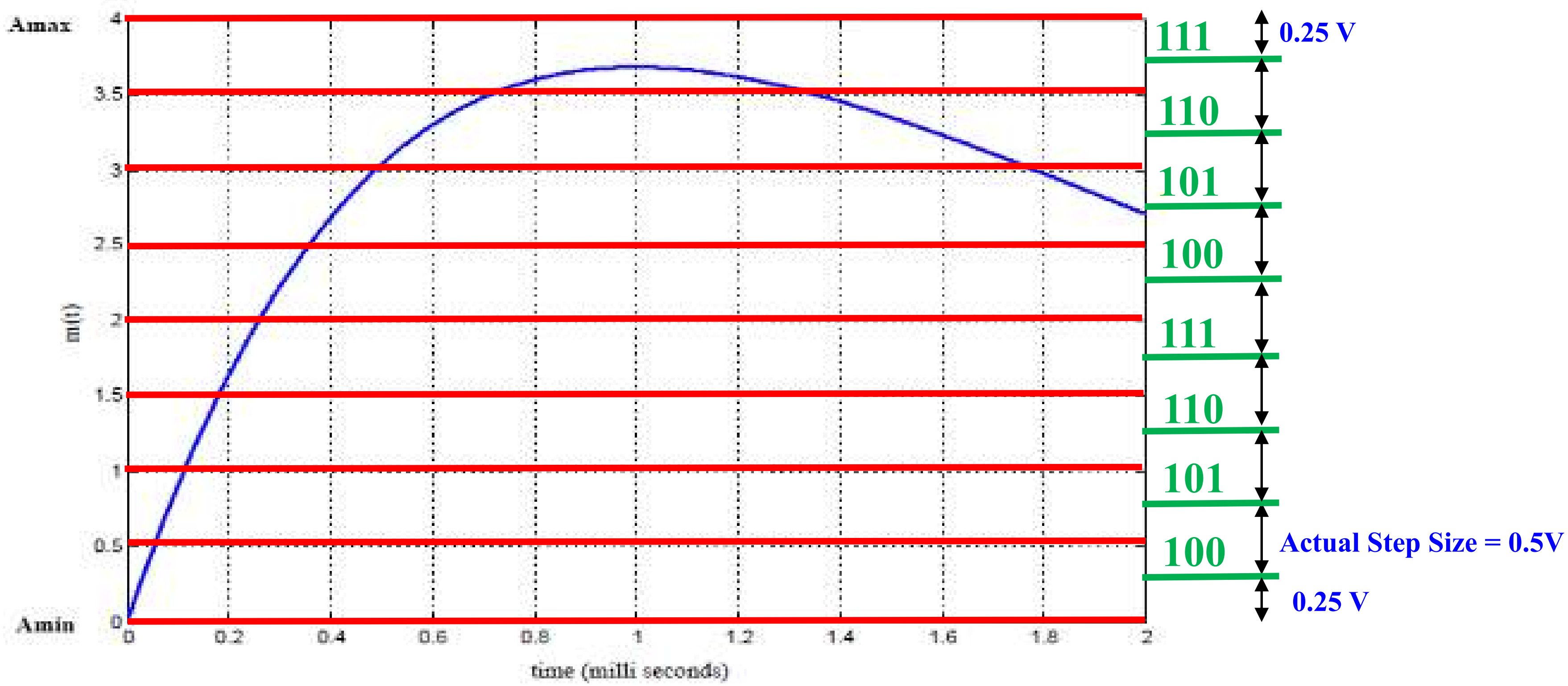
PCM – Example 2

Write the output PCM sequence using 8 levels.



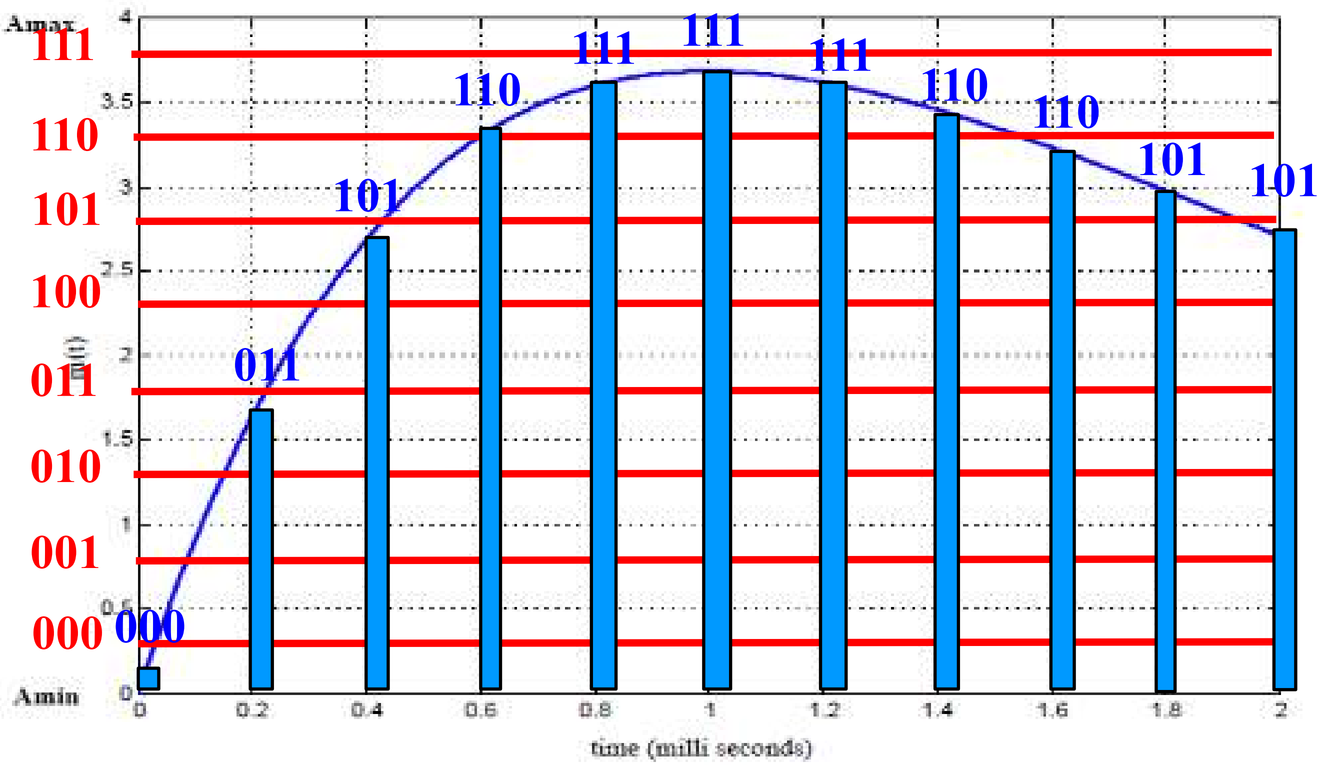
Sampling Rate = $2 \times f_{max}$ → Sampling Rate= $\frac{1}{T_s}$ → Sampling Rate= $\frac{1}{0.0002}$= 5000 Samples/sec

PCM – Example 2



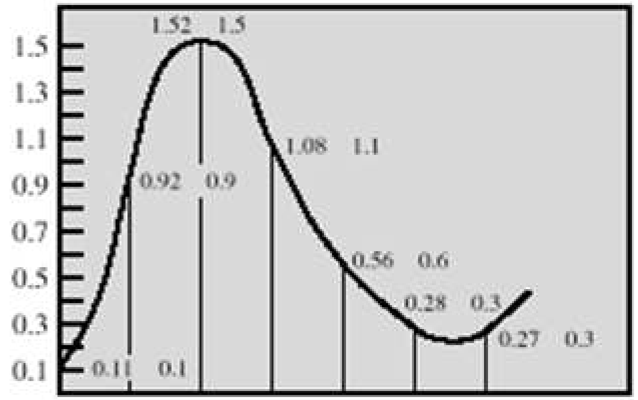
$$\textbf{Actual Resolution} = \pm\frac{\textit{Actual Step Size}}{2} = \pm 0.25\ V$$

PCM – Example 2



Output PCM sequence = 000 011 101 110 111 111 111 110 110 101 101

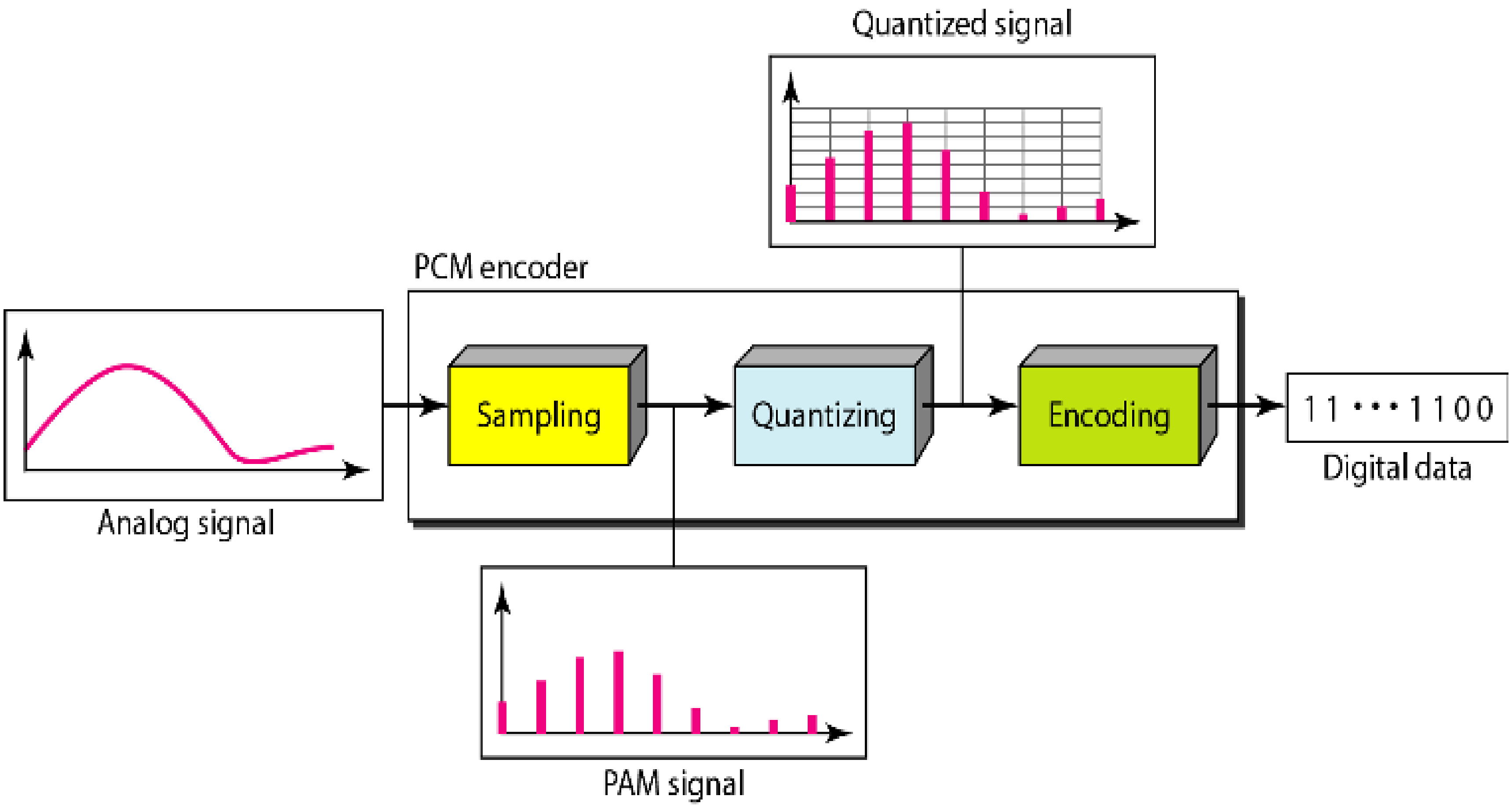
PCM



Digit	Binary Equivalent	PCM waveform
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	

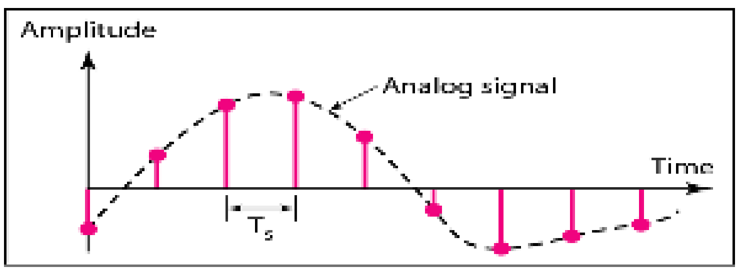
Digit	Binary Equivalent	PCM waveform
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

PCM

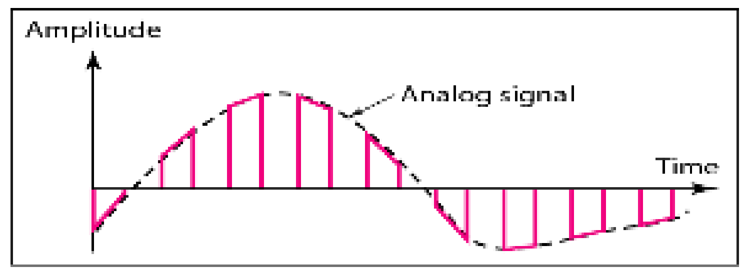

Quantized signal
PCM encoder
Sampling
Quantizing
Encoding
11 ••• 1100
Digital data
Analog signal
PAM signal

PCM

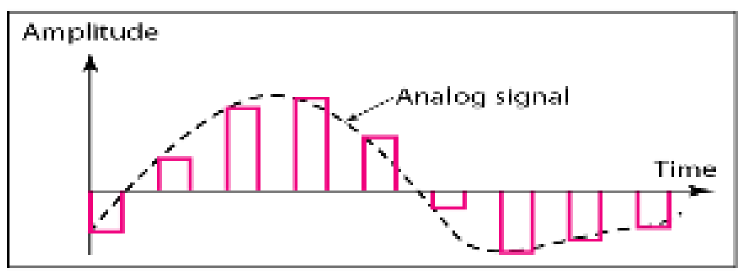
- There are **three sampling methods: ideal, natural, and flat-top.**
  - **Ideal sampling:** Pulses from the analog signal are sampled. This is an ideal sampling method and **cannot be easily implemented**.
  - **Natural sampling:** a high-speed switch is turned on for only the small period of time when the sampling occurs. **The result is a sequence of samples that retains the shape of the analog signal**.
  - **Flat-top Sampling:** The most common sampling method, called **sample and hold**.



a. Ideal sampling



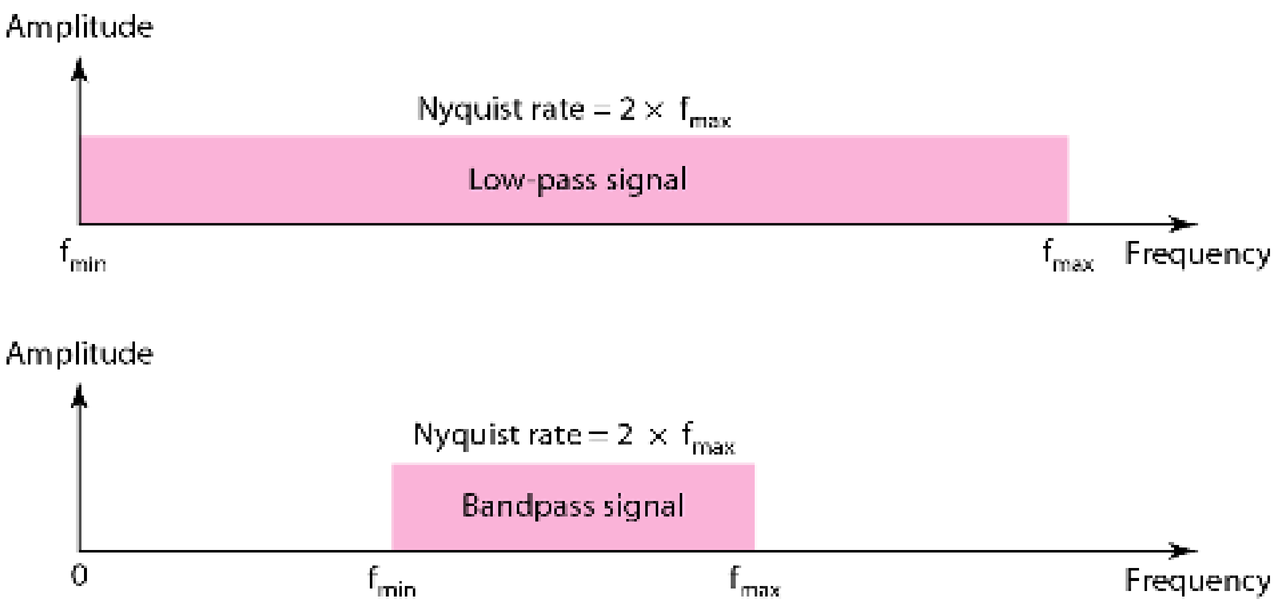
b. Natural sampling



c. Flat-top sampling

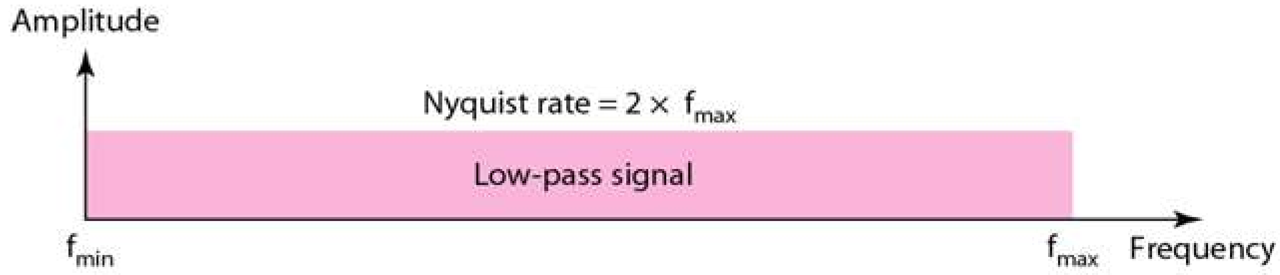
Sampling Rate

- We can sample a signal only if the signal is **band-limited. In other words, a signal with an infinite bandwidth cannot be sampled.**
- The sampling rate must be at least 2 times the highest frequency, not the bandwidth.
- If the analog signal is **low-pass**, the bandwidth and the highest frequency are the same value.
- If the analog signal is **bandpass**, the bandwidth value is lower than the value of the maximum frequency



PCM Example 1:

⌘ A complex **low-pass signal** has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

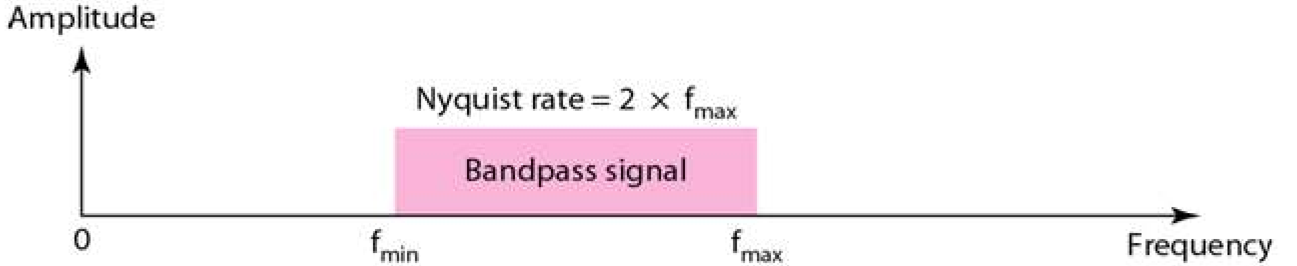


Solution

⌘ The bandwidth of a low-pass signal is between 0 and f, where f is the maximum frequency in the signal. Therefore, we can sample this signal at 2 times the highest frequency (200 kHz). **The sampling rate is therefore 400,000 samples per second**.

PCM Example 2:

⌘ A complex **bandpass signal** has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?



Solution

⌘ **We cannot find the minimum sampling rate** in this case because we do not know where the bandwidth starts or ends. **We do not know the maximum frequency in the signal.**

PCM
Example 3:

What sampling rate is needed for a signal with a bandwidth of 10000 Hz (1000 to 11000 Hz)? If the *quantization* is 8 bit per sample, what is the bit rate?

Solution:

The sampling rate must be twice of the highest frequency in the signal:

Sampling rate = 2 × 11000=22000 samples/ sec.

→ Data rate = 22000 samples/sec × 8 bit/sample =176 Kbps.

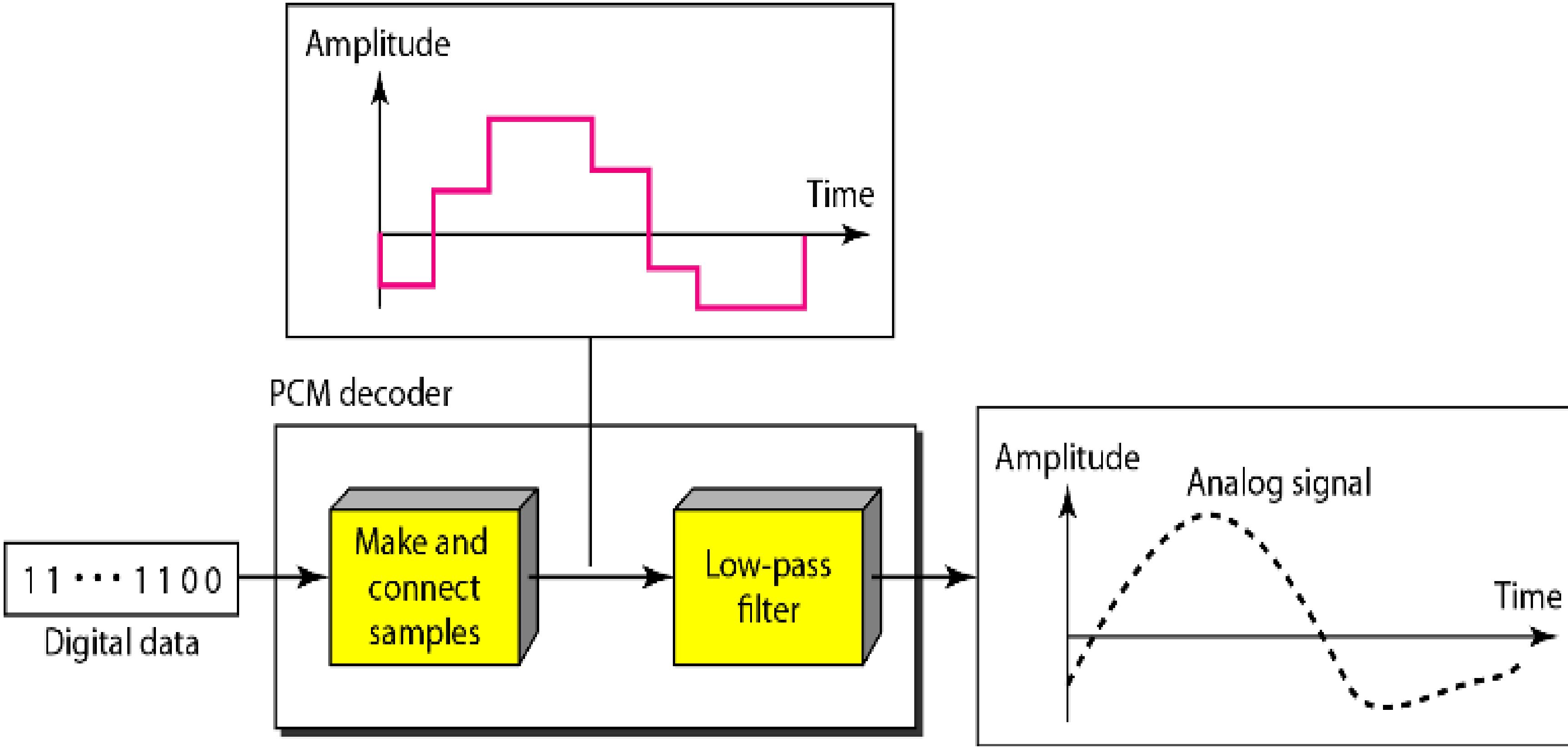
Pulse Code Modulation

- If signal with highest frequency component f_{max} is sampled at rate greater than $2f_{max}$ then samples contain all information of signal
- Sample analog signal at rate $2f_{max}$
  - called **Pulse Amplitude Modulation (PAM)**
  - $T_s = 1/\ 2f_{max}$ second
- Assign each sample a binary code
  - sample is quantized into nearest code
  - more quantization levels: better approximation
- If voice signal limited to < 4000 Hz, 8000 samples/s is sufficient

Original Signal Recovery

- The **recovery of the original signal** requires the PCM decoder.
- **The decoder first uses circuitry to convert the code words into a pulse that holds the amplitude until the next pulse.**
- **After the staircase signal is completed, it is passed through a low-pass filter to smooth the staircase signal into an analog signal.**

Original Signal Recovery



Pulse Code Modulation

- By quantizing the PAM pulse, the original signal is now only approximated and cannot be recovered exactly. This effect is known as **quantizing error** or **quantizing noise.** The **signal-to-noise ratio for quantizing noise** can be expressed as:

$$\boxed{SNR_{dB} = 20\log_{10} 2^{n} + 1.76_{dB} = 6.02n + 1.76_{dB}}$$

- Thus, each additional bit used for quantizing increases SNR by about 6 dB, which is a factor of 4.

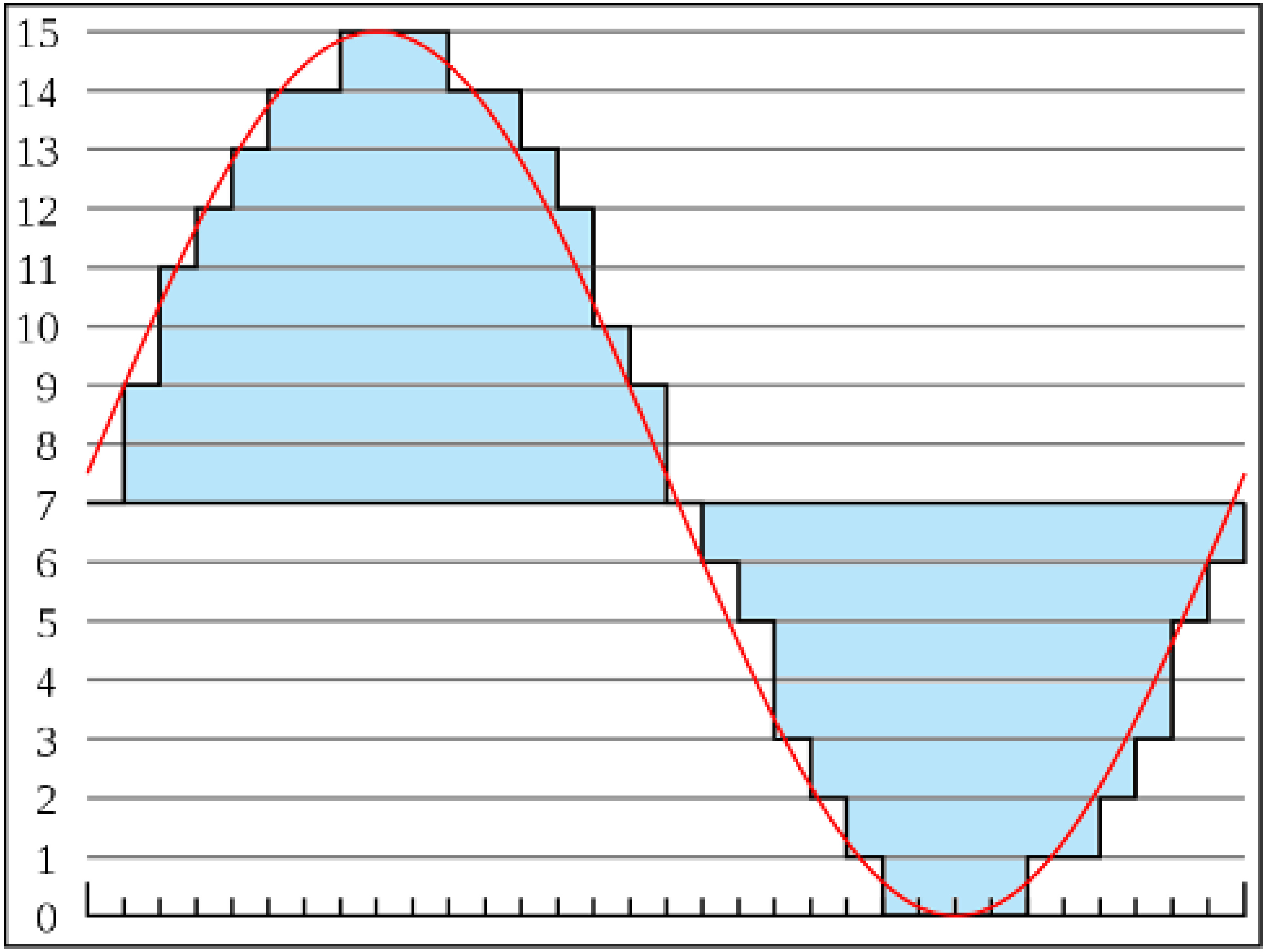
Non-Linear Encoding

- The PCM scheme is refined using a technique known as **nonlinear encoding**, which means, the quantization levels are not equally spaced.

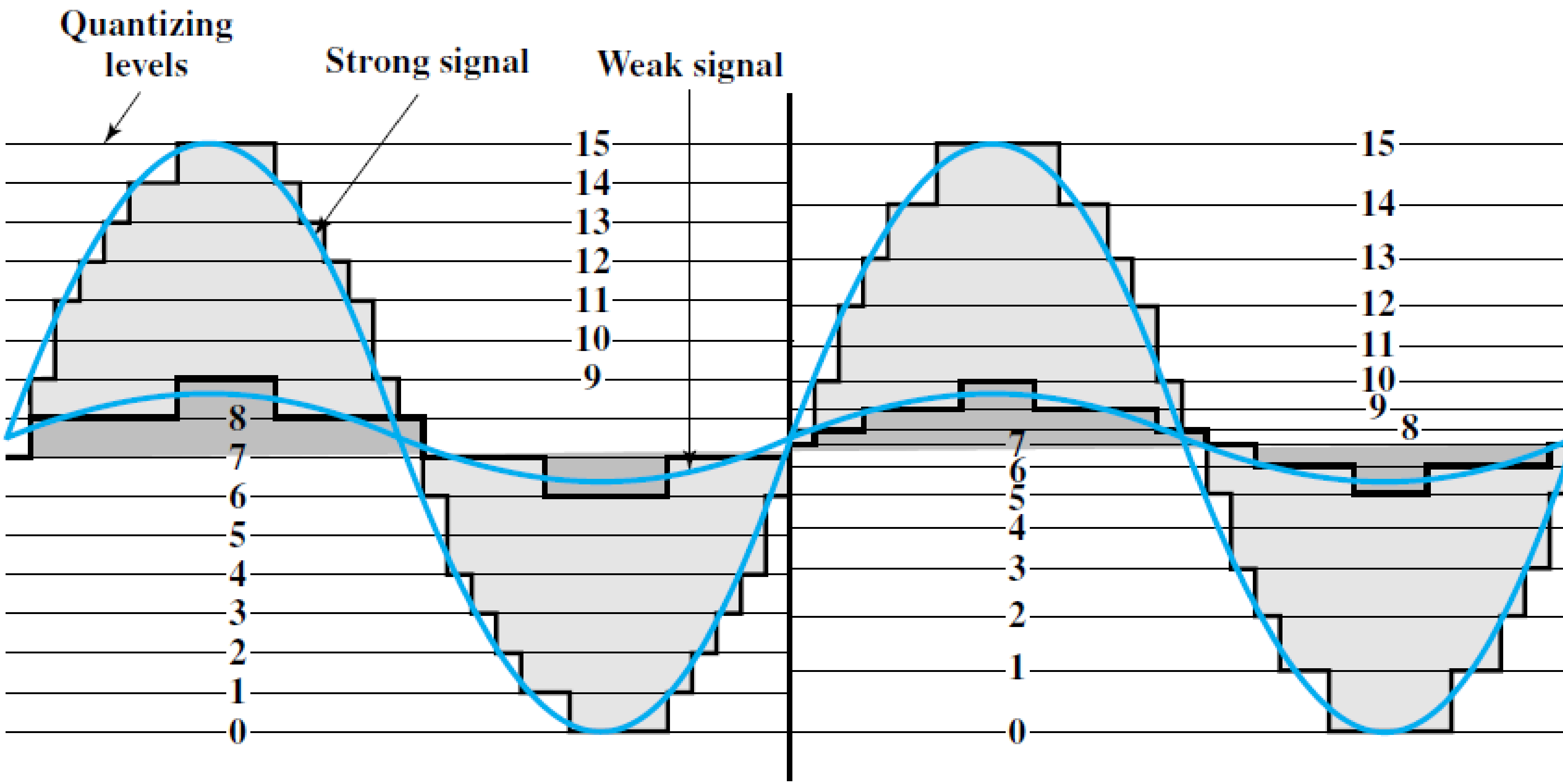
- The problem with equal spacing is that the mean absolute error for each sample is the same, regardless of signal level. Consequently, **lower amplitude values are relatively more distorted**.

- **By using a greater number of quantizing steps for signals of low amplitude, and a smaller number of quantizing steps for signals of large amplitude, a marked reduction in overall signal distortion is achieved**

Linear Encoding

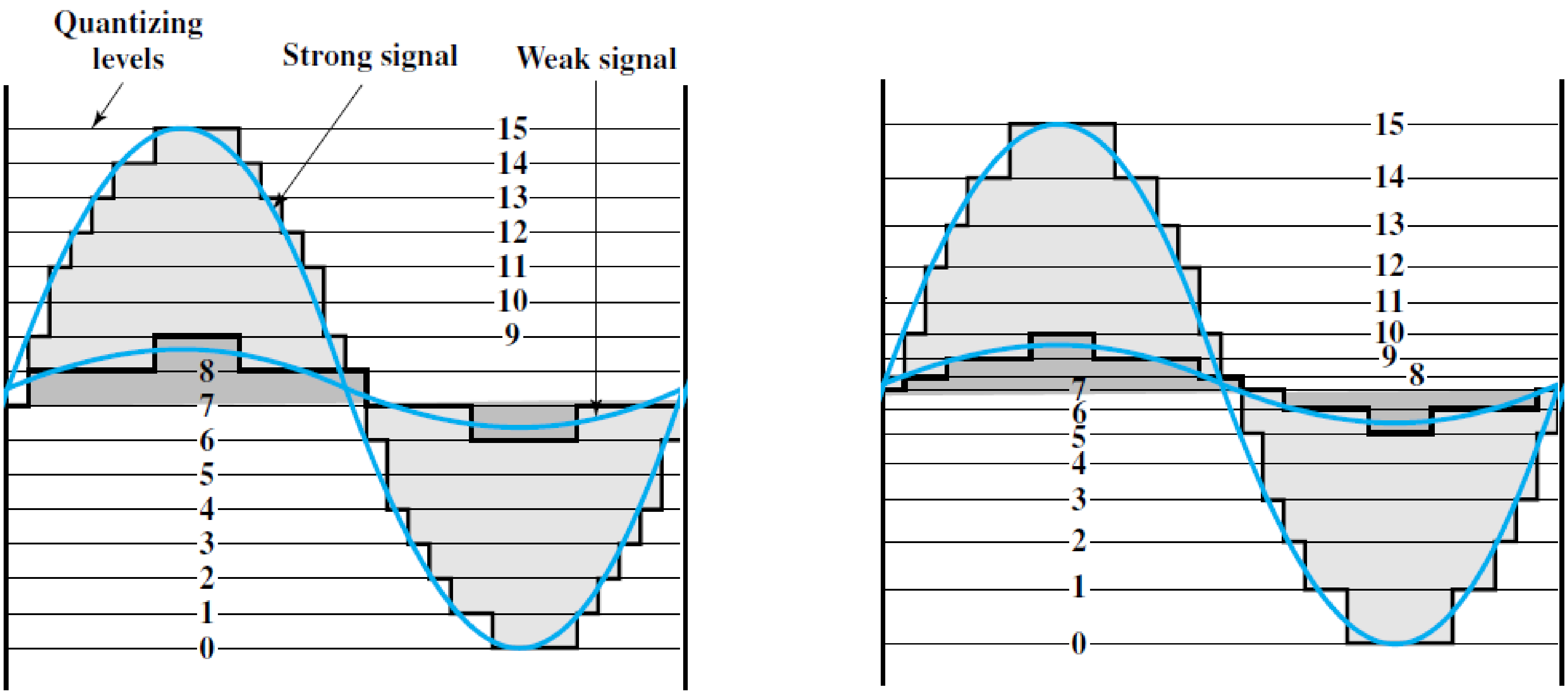


Non-Linear Encoding



(a) Without nonlinear encoding (b) With nonlinear encoding

Non-Linear Encoding



(a) Without nonlinear encoding

(b) With nonlinear encoding

Companding

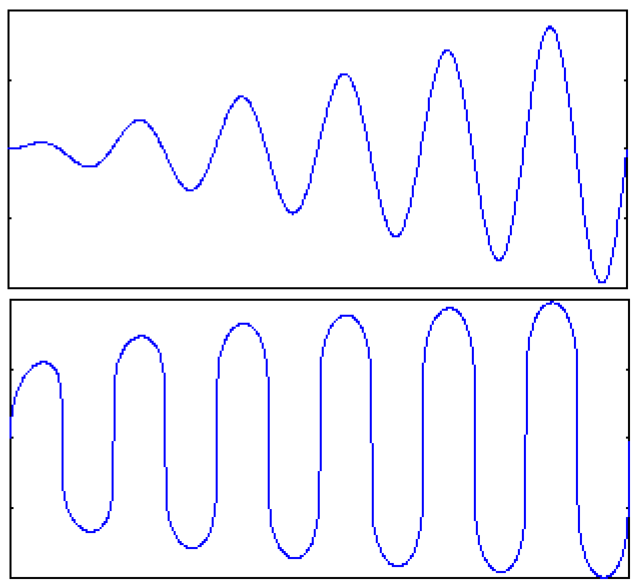
- The same effect can be achieved by using uniform quantizing but **companding (compressing-expanding) the input analog signal**.

- Companding is a process that compresses the intensity range of a signal :
  - **Input:** more gain to weak than strong signals
  - **Output:** reverse operation is performed

- The effect on the input side is to compress the sample so that the higher values are reduced with respect to the lower values. **Thus, with a fixed number of quantizing levels, more levels are available for lower-level signals. On the output side, the compander expands the samples so the compressed values are restored to their original values.**

Companding

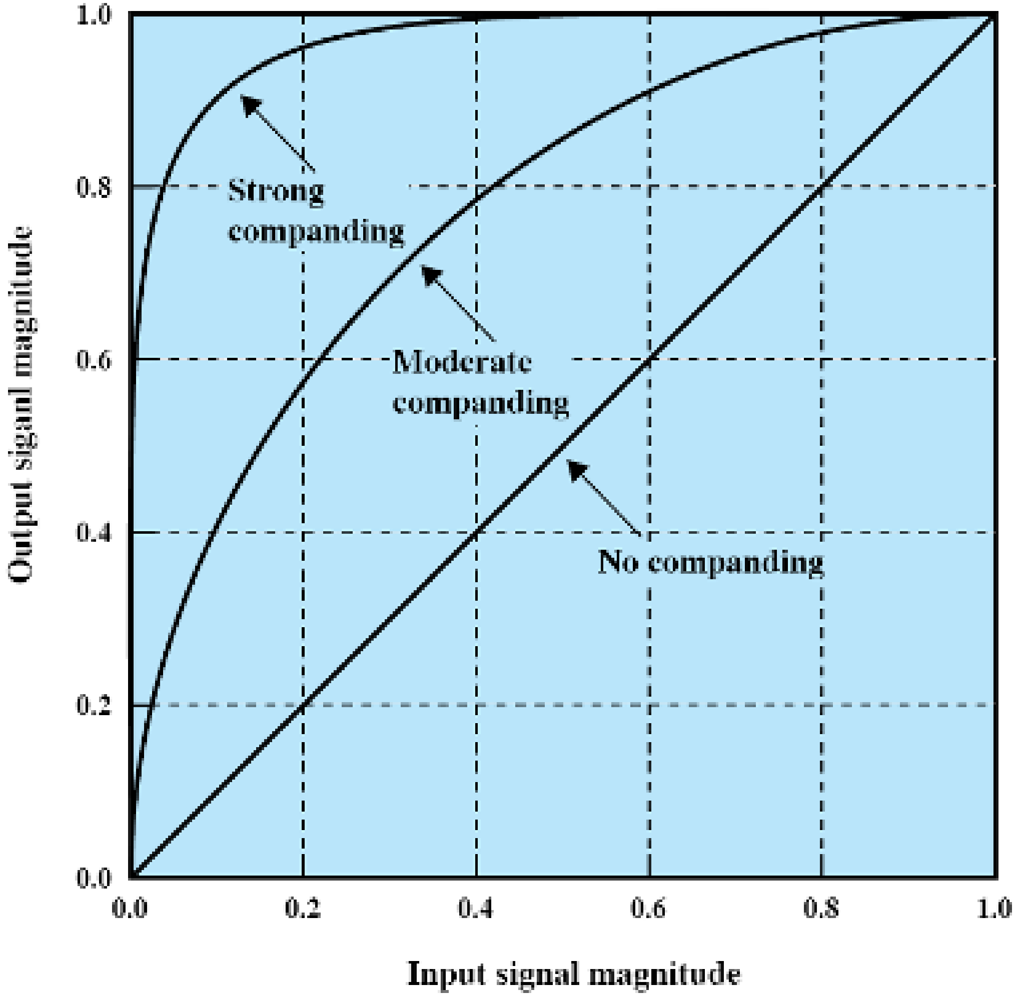
- **Companding** can refer to the use of compression, where **gain is decreased when levels rise above a certain threshold**, and its complement, expansion, where **gain is increased when levels drop bellow a certain threshold**.
- Compressing-expanding → Companding



Original Signal

After compression and before expansion

Companding

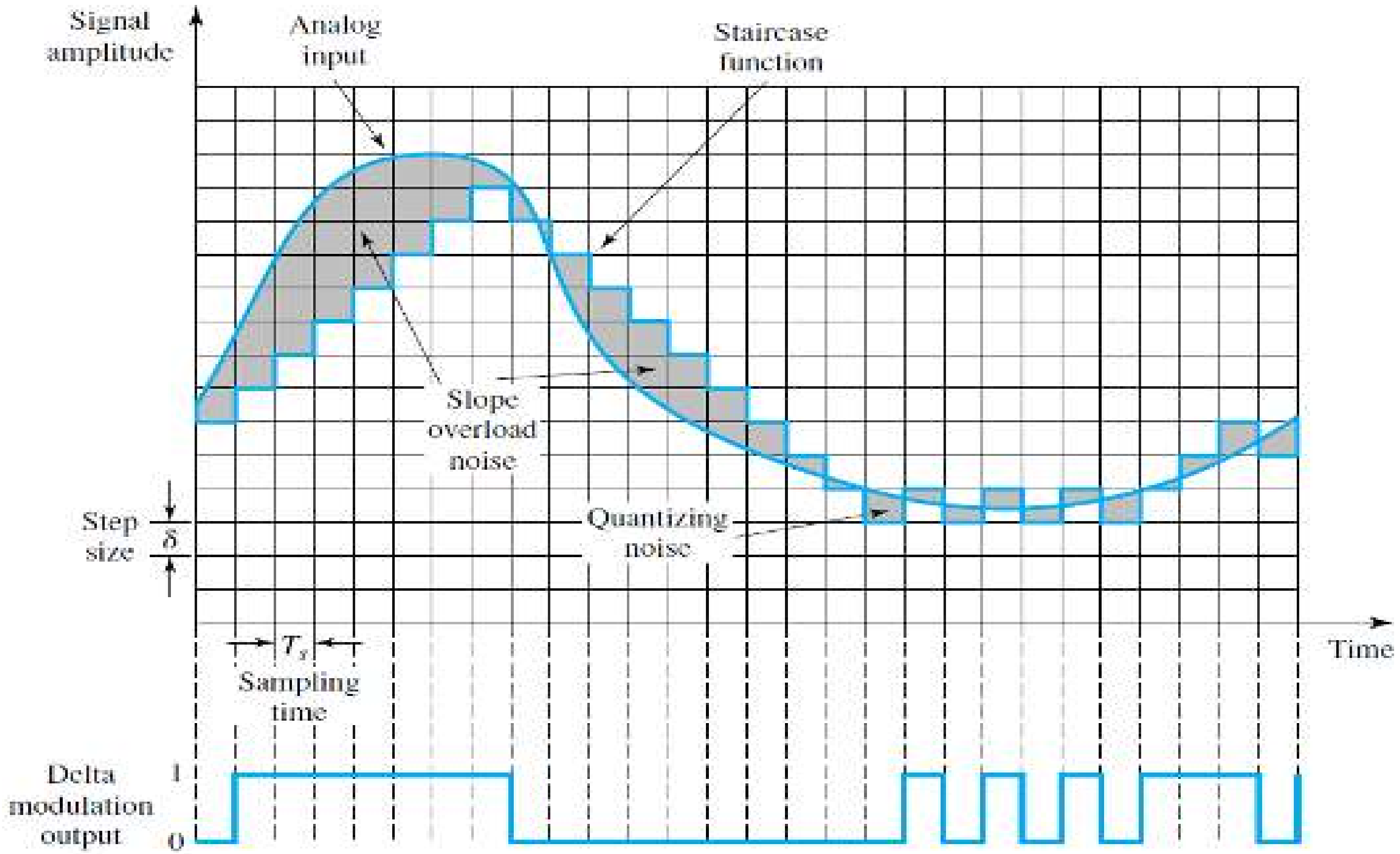

1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Output signal magnitude
Input signal magnitude
Strong
companding
Moderate
companding
No companding

Delta Modulation

- **Analog input** is approximated by a **staircase function**
  - can move up or down by one quantization level (δ) at each sampling interval (T_s)

- Step can be represented by single bit
  - **1: next interval up**
  - **0: next interval down**

Delta Modulation



Sampling Rate $= \frac{1}{T_s}$

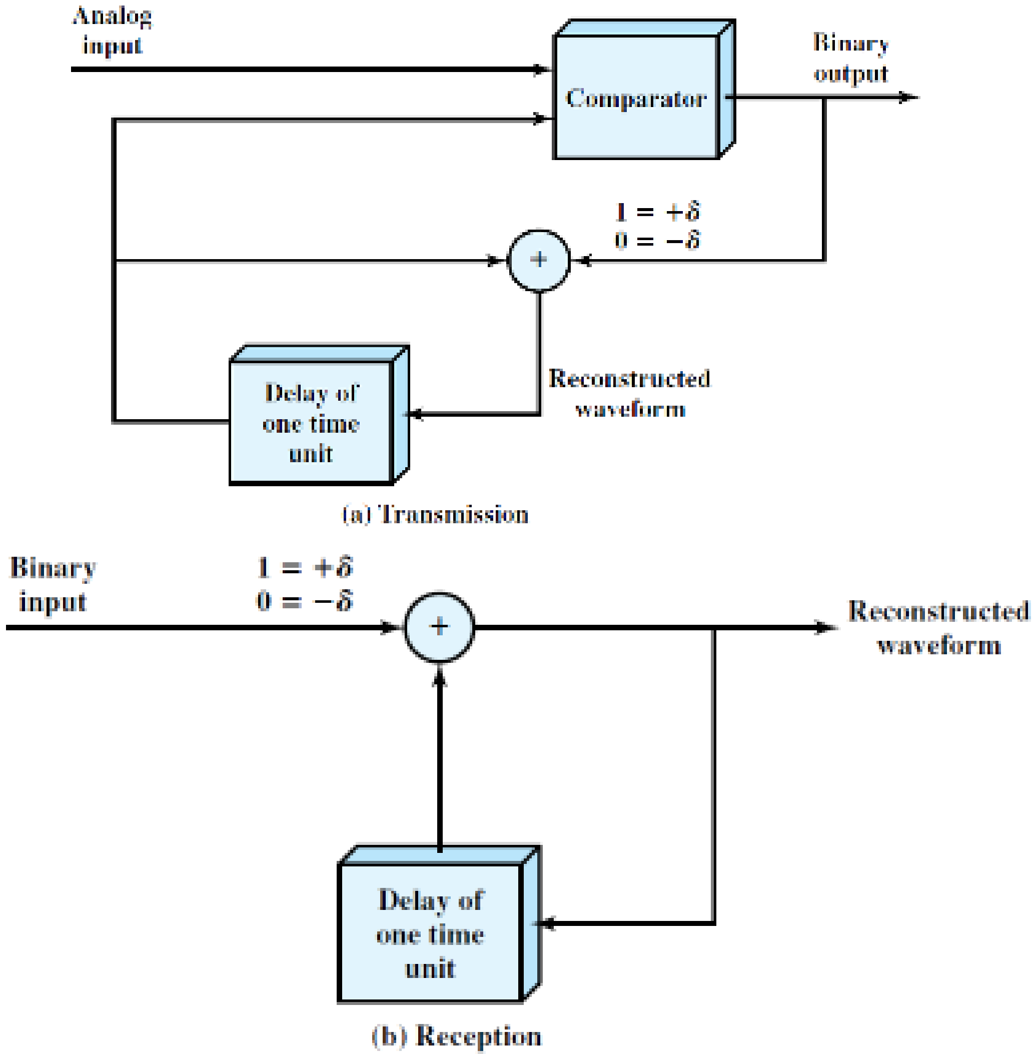
Date Rate = 1 (bit/sample) × Sampling Rate (samples/sec)

Date Rate= Sampling Rate

Delta Modulation

- **Transmission**
  - analog input A compared to latest value L of staircase function
  - **if A > L, output = 1**
  - **if A < L, output = 0**
- **Reception**
  - staircase function is reconstructed
  - **1: increment by 1 level (δ)**
  - **0: decrement by 1 level (δ)**

Delta Modulation Operation



Delta Modulation

- There are two important parameters in a DM scheme: **step size (δ)** and **sampling rate**
  - Step size δ: balance between types of errors
    - very large: There will be **quantizing noise**. The noise increases as δ is increased
    - very low: When the analog waveform is changing more rapidly than the staircase can follow, there is **slope overload noise**. This noise increases as δ is decreased
  - Sampling rate
    - higher is better, but increases data rate
- DM is simpler to implement than PCM
- PCM has better SNR characteristics at the same data rate

Analog Data, Digital Signals Performance

- issue of bandwidth used:
  - good voice reproduction with PCM
    - want 128 levels (**7 bit**) & voice bandwidth **4kHz**
    - need 8000 samples/sec x 7 bits/sample= **56kbps** digital signal
    - require **28KHz bandwidth**
    - compare to **4KHz** for analog transmission
  - even more severe with higher bandwidth signals, e.g. color television with PCM
    - 1024 quantization levels: **10-bit** coding
    - bandwidth **4.6 MHz** → **9.2 M** samples/second
    - 9.2 M × 10 = **92 Mbps**
    - require **46 MHz bandwidth**
    - compare to **4.6 MHz** for analog transmission
- yet, digital technology still preferred for transmission of analog data
- **data compression** can improve on this
  - e.g. Interframe coding techniques for video
- still growing demand for digital signals
  - use of repeaters, TDM, more efficient digital switching techniques
- **PCM preferred to DM for analog signals**

Summary

- looked at signal encoding techniques
  - digital data, digital signal
  - analog data, digital signal
  - digital data, analog signal
  - analog data, analog signal

Required Reading

- Stallings chapter 5
- Eitan Gurari, CIS 677 course notes, Ohio State University,
  www.cse.ohio-state.edu/~gurari/course/cis677/cis677Se12.html